INFLUENCE OF AN EXTERNAL MAGNETIC FIELD ON THE NANOFLUID FLOW OVER A PERMEABLE SHEET

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ABSTRACT

In this thesis, an investigation has been done regarding the magneto hydrodynamic unsteady and incompressible mixed convection nanofluid flow over a permeable sheet, utilizing the Buongiorno model. The transverse electric & magnetic field is applied in the system, whilst thermal radiation, Ohmic and viscous dissipations, heat absorption/generation effects are also considered. Governing equations with boundary layer approximation have been reduced by similarity transformations. A numerical scheme is then employed for computing the solution. The problem is studied by investigating the role of governing parameters through figures.

Keywords: External Magnetic field**,** Nanofluids, Buongiorno model, Permeable sheet, Similarity transformation, Boundary layer.

INTRODUCTON

The ordinary base fluids like oil, water and ethylene glycol, have weak heat transfer capability as compared to metals. Whereas metals have three-fold higher thermal conductivity. It was needed to form a substance which behaves like a fluid while transfers heat as a metal. Choi (1995) was the leading researcher who used the word 'nanofluid' referring to a mixture of ordinary base fluids with nanoparticles. Nanofluid is a suspension of nanoparticles in ordinary fluid and thus has a higher thermal conductivity contrary to the ordinary fluids. A nanoparticle has at least one of its dimension <100nm. Nanofluids keep enriched thermo-physical properties for example viscosity, thermal-diffusivity, convective energy-transfer coefficients and thermal-conductivity as compared to base fluids like water or oil (Minkowyez, 2012). In modern era nanofluids are used as smart fluids for heat exchange in many fields to save energy.

To achieve the stage of thermal equilibrium, heat transfers from warmer systems to cooler systems. It occurs in the modes of Conduction, Convection and Radiation. Heat transmission and cooling devices are required in numerous modern technological equipments and industrial processes like power generation plants and air conditioning or cooling plants, microelectronic devices like chips of mobile phones and processors of laptop computers to the high-tech atomic reactors and space crafts. Recent development in technology and emergence of miniaturized electrical devices, led to the need of efficient and speedy heat transfer mechanisms. The nanofluids have provided the solution to these requirements.

Different researchers have investigated the magnetic field impact on the progress of electric-conducting nanofluids like Abolbashari et al. (2014); Daniel et al. (2015a), (2015b), (2016), (2017), Hedayatnasab et al. (2017), Daniel et al. (2017a), Malvandi, Hedayati, and Ganji, (2014), Sandeep and Sulochana, (2015) and Waqas et al., (2016). Higher heat transfer and thermal conductivity are required in metallurgical processes and nuclear reactors discussed in the work of Shateyi, & Motsa (2011).

Magnetohydrodynamic (MHD) is the area of physics which studies the behavior of an electrically conducting fluid for example plasma or molten metal under the action of magnetic field. In 1970 The Nobel Prize in Physics was given to Hannes Olof G¨osta and Alfv´en for their phenomenal efforts and findings in Magnetohydrodynamic with significant uses in the area of plasma physics. Magnetic nanofluid is the suspension of magnetic particles in a suitable fluid (Daniel et al., 2017). Magneto-hydrodynamics (MHD) nanofluid has provided a mean for efficient heat transfer through convection and has got various applications in industrial science and engineering, (Linh, et al. 2015). These nanofluids have been significantly utilized in optical switches, fibers of magneto-optical wavelength, optical gratings and optical modulator. This application of nanofluids has attracted many researchers to work further in the field as did (Hayat,et al., 2016 and Bég et al., 2014). Thermal conduction has an endless significance in various industrial warming or cooling devices. Presently, the effect of magnetic nanofluids has also been studied by different analysts, for example Daniel & Daniel (2015); Daniel et al. (2017b); Freidoonimehr et al. (2015), Gómez-Pastora,et al. (2017); Hayat et al. (2011); Hayat and Qasim (2017b); Kumar, and Sood, (2017); Mabood and team (2015); Mohammed,et al. (2017) ; Shagaiy and his team (2017)).

Magneto-hydrodynamics and boundary layer flow over a stretching surface were statistically studied by Fadzilah et al. (2011) and Ishak & Nazar (2008). Their investigation demonstrated that fluid field velocity at a stage diminishes by an expansion in magnetic field by the impact of Lorentz force. Scientists, for example, Mahapatra, & Gupta (2001) and Ishak et al. (2009) broadened the element of magneto-hydrodynamics boundary power- law fluids with different fluid boundary values (Ibrahim et al., 2014).

Stratification in a fluid system means the arrangement of fluid layers due to the variances in density of fluid and the changes in their temperature and concentration. (Rehman et al. 2016).When the transfer of mass and heat occurs concurrently it is called as double stratification. Stratification of temperature differences is called thermal stratification while solutal stratification is based on concentration variations. Thermal stratification occurs in reservoirs due to density variation of water and results in reduction of vertical mixing of oxygen. The water in the bottom of reservoir becomes anoxic due to the biological processes. The commonly observed phenomenon of air density stratification is due to the temperature differences in different layers of atmospheric air. Stratification is present naturally in rivers, lakes and oceans because of environmental differences and temperature variation. Thus we can say that stratification occurs in fluid flow due to the variation in its density, and because of the changes in concentration and temperature. An example of this phenomenon is the variation in density of air and water due to the differences in their temperature at different heights or levels, causing change in their movement or flow (Kandasamy et al.2016).

On micro-level the effect of solutal stratification and thermal stratification on mass and heat transmission investigation in boundary layer nanofluid flow above a stretching surface has fundamental importance in engineering and manufacturing processes like manufacturing of sheets of rubber and plastic, copper wires thinning and annealing, refrigeration and air-cooling system, compact heat transfers, solar energy collectors etc (Sajid et al. 2010; Hayat et al. 2012).

This phenomenon of double stratification with convective heat flow has been studied by different researchers like Cheng and Lee (2009), Cheng (2008), Ibrahim and Makinde (2013).

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Stratification is a significant phenomenon pertaining to heat and mass transport processes. Due to its importance in manufacturing processes, many researchers have studied this phenomenon. In this connection, Ahmad et al. (2018) considered mixed convection characteristics of sutterby fluid flow passing by squeezed channel and proved that the mass and heat transfer rate declines with foremost double stratification parameter and chemical reaction. Rehman et al. (2017) studied dual stratification due to differences in concentration in chemically reactive substances in fluid flowing on an inclined cylindrical stretching surface. Hayat et al., (2016a) uncovered features of heat-conductivity on double stratified flow of fluid deformed by permeable surface and Kandasamy et al. (2016) depicted in Magnetohydrodynamic flow of double stratified nanofluid passed by a pored surface. Rehman et al. (2016) researched the properties of heat -source and heat-sink on convective flow of Powell–Eyring fluid through shrinking cylinder and double stratification. Two-fold stratification (thermal stratifications and mass stratification) on the convective heat transportation in nanofluids, have been studied by Hayat et al. (2016d), Hayat et al. (2017a), Mehmood, Hussain, and Sagheer (2016) and Ramzan et al. (2017). The figure 1 below, shows the magnetic field generated through electrical current and the resultant magnetic force and its direction.

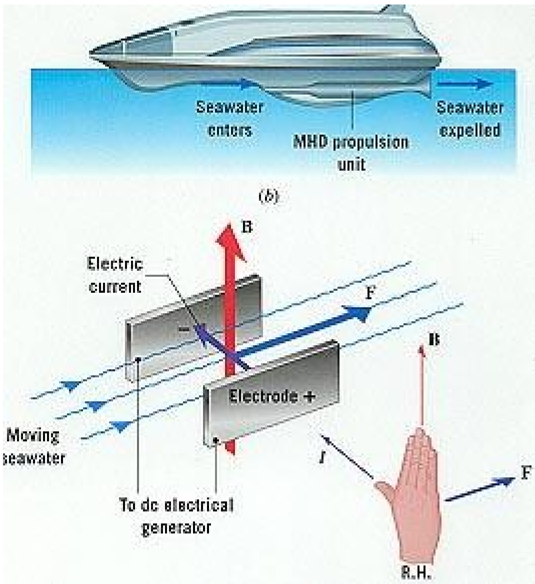

Figure 1: Water boat made on the principle of MHD (Jabeen et al. 2019)

Lugwid Prandtl introduced the concept of boundary layer in 1904. It is the fluid next to the surface of a body that effects the velocity of fluid by its viscosity. From initial point where no velocity is present up till the velocity where it is almost equal to the 99% of normal flow is known as a thickness of boundary layer.

Application of the boundary layer theory in the industrial processes like airfoil designing for airplanes, friction drag of a ship, and many other procedures has been a main cause of curiosity among researchers. Crane and Miklavˇciˇc (1975) and Wang (1995) were some of the early researchers worked on this subject. Khashi'ie et al., 2019 is a good source for studying the historical developments in this area.

The boundary layer flow over an extending surface with mass and heat exchange has application in manufacturing of paper, glass fiber-production, plastic films and metal plates. In this manufacturing process of sheets, through slit a melting matter comes out and then stretched to the required thickness. The ultimate manufactured goods with its final features depends on rate of stretching and rate of cooling. During this process of stretching the nanofluids provide efficient cooling as compared to the conventional fluids. Ibrahim & Shankar (2013a, 2013b and 2014) have studied boundary layer flow of double stratified over vertical plate, MHD flow past a permeable sheet and also over non-isothermal stretching sheet, while Khan et al. (2017) studied the MHD boundary layer thermal slip stream with suction-injection on nonlinear stretching cylinder, Mabood et al. (2015) analyzed it over a nonlinear stretching sheet etc.

Brownian motion is the irregular movement of nanoparticles within a base fluid. This motion is the result of regular collision among the molecules of fluid and nano-sized particles. By the impact of stable temperature gradient and Brownian motion, thermopherosis process occurs, which possess a great significance in nanofluids flow.

To investigate the transport of thermal energy, Buongiorno (2006) built a two-phase model under the influence of thermopherosis and Brownian motion for nanofluid which has been effectively used to address different assorted flow problems. Hayat .et al. (2016b), Mabood and Khan, (2016)) worked on the aspect of thermophoretic diffusion and Brownian motion using Buongiorno model.

Time dependent problems of unsteady heat exchange have been considered earlier by Anderson, Aarseth, and Dandapat (2000). They devised the similarity resolution for temperature field which transfers the time dependent thermal equation of energy to ODE. Elbashbeshy et al. (2004) searched the heat transmission over an unsteady spreading surface. Heat transfer over unsteady vertical extending surface was considered to study by Ishak et al. 2008; Ishak et al. (2009) and Reddy et al. (2015b) have additionally examined the unsteady laminar boundary layer over a constantly extending surface.

The current work aims to study the nanofluid with the presence of thermal and concentration stratification along with the impacts of M (the magnetic field), Rd (the thermal radiation), γ the chemical reaction, ε heat absorption/generation for unsteady electrical conducting mixed convection flow considering along joule heating and viscous dissipation over a permeable stretching sheet. Buongiorno nanofluid model has been used for incorporating the particular features of the considered forces and their impact on the nanofluids and boundary layer thickness. Influence of these parameters on velocity-temperature-concentration have been discussed extensively.

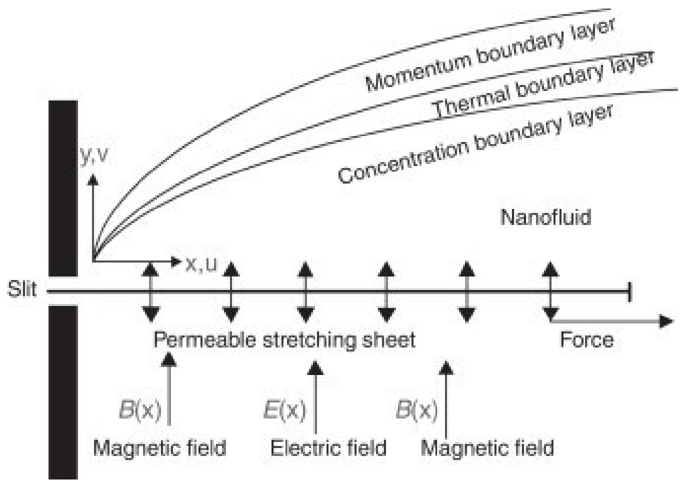

Figure 2: Graphical description of the case under consideration

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2. MATHEMATICAL FORMULATION

We assumed the unsteady, incompressible flow of an electrically conducting nanofluid past a linearly stretching surface. Further, viscous dissipation and joule heating are also taken into account. The cartesian coordinates scheme is chosen as x_1-axis is along the sheet and the x_2-axis is normal to the surface as in Figure 1.The nanofluid is considered to be conducting electrically through a slit above the surface of the sheet along x_1-axis, and is under the effect of a uniform magnetic field having power B_0, directed in the transverse axis. The flow is thermally radiative because of the stretching of the sheet through a couple of equal and reverse forces. Impacts of thermal generation/absorption are also measured in the model.

3. GOVERNING EQUATIONS

Physical description and the mathematical model (the equations of continuity, momentum, energy and concentration) under the assumption afore-mentioned are:

$$\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}=0 \qquad [1]$$

$$\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=-\frac{1}{\rho_f}\frac{\partial P}{\partial x_1}+\nu\left(\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_1}{\partial x_2^2}\right)+\frac{\sigma}{\rho_f}\left(EB-B^2u_1\right)+\frac{1}{\rho_f}\left[(1-\varphi_\infty)\rho_{f\infty}\beta\left(\rho_{p-\rho_{f\infty}}\right)\beta(\varphi-\varphi_\infty)\right]g \qquad [2]$$

$$\frac{\partial u_2}{\partial t}+u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=-\frac{1}{\rho_f}\frac{\partial P}{\partial x_2}+\nu\left(\frac{\partial^2 u_2}{\partial x_1^2}+\frac{\partial^2 u_2}{\partial x_2^2}\right)+\frac{\sigma}{\rho_f}\left(EB-B^2u_2\right)+\frac{1}{\rho_f}\left[(1-\varphi_\infty)\rho_{f\infty}\beta\left(\rho_{p-\rho_{f\infty}}\right)\beta(\varphi-\varphi_\infty)\right]g \qquad [3]$$

$$\frac{\partial T}{\partial t}+u_1\frac{\partial T}{\partial x_1}+u_2\frac{\partial T}{\partial x_2}=\frac{k}{(\rho c)_f}\left(\frac{\partial^2 T}{\partial x_1^2}+\frac{\partial^2 T}{\partial x_2^2}\right)-\frac{1}{(\rho c)_f}\left(\frac{\partial q_r}{\partial x_2}\right)+\frac{\mu}{(\rho c)_f}\left(\frac{\partial u_1}{\partial x_2}\right)^2+\frac{Q}{(\rho c)_f}(T-T_\infty)+\frac{\sigma}{(\rho c)_f}(u_1B-E)^2$$
$$+\tau\left\{D_B\left(\frac{\partial\varphi}{\partial x_1}\frac{\partial T}{\partial x_1}+\frac{\partial\varphi}{\partial x_2}\frac{\partial T}{\partial x_2}\right)+\frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x_1}\right)^2+\left(\frac{\partial T}{\partial x_2}\right)^2\right]\right\} \qquad [4]$$

$$\frac{\partial\varphi}{\partial t}+u_1\frac{\partial\varphi}{\partial x_1}+u_2\frac{\partial\varphi}{\partial x_2}=D_B\left(\frac{\partial^2\varphi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_2^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x_1^2}+\frac{\partial^2 T}{\partial x_2^2}\right)-k_1(\varphi-\varphi_\infty) \qquad [5]$$

Here u_1 and v_1 are the velocity components in the x_1 and x_2 directions respectively, ν is the kinematic viscosity of the fluid, P is the pressure, B_0 is the magnetic field, B is the applied magnetic field, σ is the electric conductivity, E is the applied electric field, g is the magnitude of the gravity, $(\rho c)_f$ is the heat capacity of the fluid, k is the thermal conductivity, β is the volume expansion coefficient, β_φ and β_T are the coefficient of thermal and mass diffusion, φ_w and φ_∞ are the values of the volume nanoparticles at the surface and above the surface, values of x_2, ρ_f is the fluid density, q_r is the radiative heat flux, Q is the heat absorption/generation , k_1 is the rate of chemical reaction, τ is the relation amongst the effective heat transfer ability of the ultra-fine nanoparticles material and thermal capacity of the fluid, D_B and D_T are the coefficients of Brownian diffusion and the coefficient of thermophoresis diffusion, T and φ are the fluid temperature and concentration.

The suitable boundary conditions for the current investigation are:

$$\left.\begin{aligned} &u_1 = u_w(x_1,t) = bx_1(1-at)^{-1},\\ &u_2 = v_w(x_1,t),\\ &T = T_w(x_1) = T_0 + A_1x_1(1-at)^{-1},\\ &\varphi = \varphi_w(x_1) = \varphi_0 + C_1x_1(1-at)^{-1}, \end{aligned}\right\} \quad at \quad x_2 = 0,$$

$$\left.\begin{aligned} &u_1 \to 0,\\ &T \to T_\infty = T_0 + A_2x_1(1-at)^{-1},\\ &\varphi = \varphi_\infty = \varphi_0 + C_2x_1(1-at)^{-1} \end{aligned}\right\} \quad at \quad x_2 \to \infty, \qquad [6]$$

Where b is the constant stretching rate of the sheet, and v_w is the normal velocity component at the surface, A_1, A_2, C_1 and C_2 are the dimensional constants.

The radiative heat flux q_r by Rosseland estimation, after some manipulations, is given as

$$\frac{\partial q_r}{\partial x_2} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial x_2^{\,2}} \qquad [7]$$

where σ^* is the Stefan– Boltzmann constant and k^* is the mean absorption coefficient. Replacing Eq. [7] by Eq. [4], has change into following

$$\begin{aligned} &\frac{\partial T}{\partial t} + u_1\frac{\partial T}{\partial x_1} + u_2\frac{\partial T}{\partial x_2} = \frac{k}{(\rho c)_f}\left(\frac{\partial^2 T}{\partial x_1^{\,2}} + \frac{\partial^2 T}{\partial x_2^{\,2}}\right) + \frac{1}{(\rho c)_f}\left(\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial x_2^{\,2}}\right) +\\ &\frac{\mu}{(\rho c)_f}\left(\frac{\partial u_1}{\partial x_2}\right)^2 + \frac{Q}{(\rho c)_f}(T - T_\infty) + \frac{\sigma}{(\rho c)_f}(u_1B - E)^2 + \\ &\tau\left\{D_B\left(\frac{\partial \varphi}{\partial x_1}\frac{\partial T}{\partial x_1} + \frac{\partial \varphi}{\partial x_2}\frac{\partial T}{\partial x_2}\right) + \frac{D_T}{T_\infty}\left[\left(\frac{\partial T}{\partial x_1}\right)^2 + \left(\frac{\partial T}{\partial x_2}\right)^2\right]\right\} \end{aligned} \qquad [8]$$

Utilizing the order of magnitude investigation for the equation of momentum in x_2- direction which is perpendicular to the sheet and applying the approximation of boundary layer

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$$\left.\begin{array}{l} u_1 >> u_2 \\ \dfrac{\partial u_1}{\partial x_2} >> \dfrac{\partial u_1}{\partial x_1}, \dfrac{\partial u_2}{\partial t}, \dfrac{\partial u_2}{\partial x_1}, \dfrac{\partial u_2}{\partial x_2} \\ \dfrac{\partial p}{\partial x_2} = 0 \end{array}\right\} \quad [9]$$

Subsequently, the implication of boundary layer approximation, the Eqs. [1],[3], [5]and [8] are reduced into

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} = 0, \quad [10]$$

$$\begin{gathered} \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x_1} + u_2 \frac{\partial u_1}{\partial x_2} = \nu \left(\frac{\partial^2 u_1}{\partial x_2{}^2} \right) + \frac{\sigma}{\rho_f} \left(EB - B^2 u_1 \right) \\ + \frac{1}{\rho_f} \left[(1-\varphi_\infty) \rho_{f\infty} \beta (T - T_\infty) - (\rho_p - \rho_{f\infty}) \beta (\varphi - \varphi_\infty) \right] g, \end{gathered} \quad [11]$$

$$\begin{gathered} \frac{\partial T}{\partial t} + u_1 \frac{\partial T}{\partial x_1} + u_2 \frac{\partial T}{\partial x_2} = \frac{k}{(\rho c)_f} \left(\frac{\partial^2 T}{\partial x_2{}^2} \right) + \frac{1}{(\rho c)_f} \left(\frac{16 T_\infty^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial x_2{}^2} \right) + \\ \frac{\mu}{(\rho c)_f} \left(\frac{\partial u_2}{\partial x_2} \right)^2 + \frac{Q}{(\rho c)_f} (T - T_\infty) + \frac{\sigma}{(\rho c)_f} (u_1 B - E)^2 + \\ \tau \left\{ D_B \left(\frac{\partial \varphi}{\partial x_2} \frac{\partial T}{\partial x_2} \right) + \frac{D_T}{T_\infty} \left[\left(\frac{\partial T}{\partial x_2} \right)^2 \right] \right\}, \end{gathered} \quad (2.12)$$

$$\frac{\partial \varphi}{\partial t} + u_1 \frac{\partial \varphi}{\partial x_2} + u_2 \frac{\partial \varphi}{\partial x_2} = D_b \left(\frac{\partial^2 \varphi}{\partial x_2{}^2} \right) + \frac{D_t}{T_\infty} \left(\frac{\partial^2 T}{\partial x_2{}^2} \right) - k_1 (\varphi - \varphi_\infty), \quad (2.13)$$

To achieve the similarity solution of the Eqs. [10]-[13], we utilize the following non-dimensional variables:

$$\begin{gathered} \psi = \sqrt{\frac{b\nu}{1-at}} x_1 f(\eta), \qquad \eta = x_2 \sqrt{\frac{b}{\nu(1-at)}}, \\ \theta = \frac{T - T_\infty}{T_w - T_0}, \qquad \phi = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_0}. \end{gathered} \quad [14]$$

Whereas the stream function ψ is described as

$$u_1 = \frac{\partial \psi}{\partial x_2}, \qquad u_2 = -\frac{\partial \psi}{\partial x_1}. \quad [15]$$

Finally, Eqs. [10]-[13] are changed to

$$f''' + ff'' - f'^2 - \delta\left(f' + \frac{\eta}{2}f''\right) + M\left[E_1 - f'\right] + \lambda\left(\theta - N_r\phi\right) = 0 \qquad [16]$$

$$\frac{1}{P_r}\left(1+\frac{4}{3}R_d\right)\theta'' + f\theta' - f'\theta - \delta\left(s_t + \frac{\eta}{2}\theta' + \theta\right) + N_b\phi'\theta' + N_t\theta'^2 + E_c(f'')^2 + ME_c(f'+E_1)^2 + \varepsilon\theta + s_t f' = 0 \qquad [17]$$

$$\phi'' + L_e f\phi' - L_e f'\phi - L_e\delta\left(s_c + \frac{\eta}{2}\phi' + \phi\right) + \frac{N_t}{N_b}\theta'' - L_e\gamma\phi - L_e s_c f' = 0 \qquad [18]$$

Where $\delta = a/b$ is unsteady parameter, $\lambda = G_r/R_e^2$ is the mixed convection, $G_r = g\beta(1-\varphi_\infty)(T_w - T_\infty)\rho_{f\infty}/v^2\rho_f$ is the Grashof number, $\mathrm{Re} = u_w x/v$ is the Reynold number, $P_r = v/\alpha$ is the Prandtle number, $N_r = (\rho_p - \rho_{f\infty})(\varphi_w - \varphi_0)/\beta\rho_{f\infty}(1-\varphi_\infty)(T_w - T_0)$ is the buoyancy ratio, $N_b = (\rho c)_p D_B(\varphi_w - \varphi_\infty)/(\rho c)_f v$ is the parameter for Brownian motion, $L_e = v/D_B$ is the Lewis number, $N_t = (\rho c)_p D_T(T_w - T_\infty)/(\rho c)_f vT_\infty$ is the thermophoresis parameter, $M = \sigma B_0^2/b\rho_f$ is the magnetic field, $E_1 = E_0/u_w B_0$ is the electric field, $E_c = u_w^2/c_p(T_w - T_\infty)$ is the Eckert number, $s_c = C_2/C_1$ is the parameter of stratification , $\varepsilon = Q_0/b(\rho c)_f$ is the heat absorption and generation, $R_d = 4\sigma * T_\infty^3/k*k$ is the radiation parameter and $\gamma = k_0/b$ is the chemical reaction.

4. BOUNDARY CONDITIONS

The transformed boundary conditions are:

$$\left.\begin{array}{l} f = s, \\ f' = 1, \\ \theta = 1 - s_t, \\ \phi = 1 - s_c, \end{array}\right\} \quad at \quad \eta = 0,$$

$$\left.\begin{array}{l} f' = 0 \\ \theta = 1, \\ \phi = 0, \end{array}\right\} \quad at \quad \eta = \infty. \qquad [19]$$

5. RESULTS & DISCUSSION

In this research, finite difference method has been used for the mathematical computation of the governing equations, under the boundary conditions given in Eq. [19]. The computational investigation is performed with the parameters, Pr ,M, $E_1, \delta, \lambda\ Rd, s, N_t, N_b, E_c, \varepsilon, Le, S_t, S_c, \gamma$ i.e. Prandtl-number, the magnetic parameter, the electric field, mixed convection parameter, unsteadiness parameter, thermal radiation parameter, the buoyancy ratio parameter,

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suction/infusion, thermophoresis, the Brownian motion, , the Eckert number , heat ingestion/generation, the Lewis number, thermal stratification, concentration stratification and the chemical reaction parameters, respectively. We have selected a step size as $\Delta\eta = \frac{1}{1000}$ for numerical solution, and the tolerance is taken as 10^{-5}. The outcomes of the study is presented in graphs for f the function, f' the velocity, q the temperature, and f profiles of concentration. Figure 3 clearly showing the velocity of nanofluid is rising with the increase of electric field parameter. The electric parameter is acting as an accelerating force here causing increase in velocity and temperature and enhancing concentration, hence decreasing momentum boundary layer thickness. Higher the electric parameter, the higher the Lorentz force and vice versa. And this enhanced Lorentz force resolves the sticky effect due to the presence of nanoparticles. In Figure 4-6.The velocity and momentum boundary layer thickness behave inversely with higher magnetic parameters while it is directly proportional in the case of temperature. Lorentz force causing resistance of fluid flow and reduction of velocity and boundary layer thickness whilst nanofluid temperature is boosted by it. The increasing values of the suction parameter $(s > 0)$ decreasing the momentum boundary layer thickness of the nanofluid in Figure 7. When heat surface is positive i.e. $(\lambda > 0)$, fluid velocity and thickness of boundary layer momentum enhanced shown in Figure 8. causing fluid to flow in upward direction whereas at $(\lambda < 0)$ nanofluid flows in downward direction and $(\lambda = 0)$ shows an absence of mixed convection. Raising the parameters of the Brownian motion, decreases the temperature & concentration in figure 9 & 10 respectively. It is clear in Figure 11 that due to increase in the thermophoresis parameters, concentration is decreasing. Figure 12 illustrating the impact of the Eckert number Ec on thermal boundary layer thickness which is rising because of Eckert numbers. Thermal radiation improves the impact of nanofluid by increasing its temperature. Moreover, by increasing the thermal radiation, thermal boundary layer thickness increases as it is obvious by Figure 13. The nanofluid velocity and temperature is diminished for higher estimations of stratification of heat. It is noticeable in Figure 14 that the state of prescribed surface temperature is accomplished when thermal stratification does not exist at ($s_t = 0$). Physically, a high measure of thermal stratification varies among surface .Thusly, derives lower value of fluid temperature of nanofluid at boundary layer and reduced the depth of thermal boundary layer. Here reverse observation is noticed with the boundary layer thickness due to solutal concentration in Figure 15. It is a consequence of the fact that the fluid closer to plate have a lower concentration than the encompassing medium but it is maximum at $(s_c = 0)$. It is valued-mentioning that the nanoparticles concentration decreased altogether with higher values of Lewis number. This decrease in nanoparticles concentration is because of the modification in coefficient of Brownian motion. Higher estimations of Lewis number result in the reduced coefficient of Brownian motion illustrated in Figure 16. Figure 17 highlights the consequence of the chemical reaction parameter γ on thermal profile. The parameter enhances the profile for its positive values (generative chemical reaction). However, an opposite trend may be seen for its negative (destructive chemical reaction).

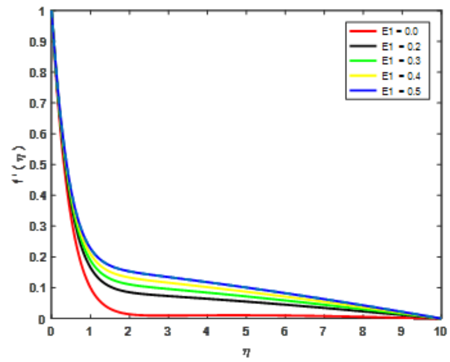

Figure 3: f' profile with the impact of E_1.

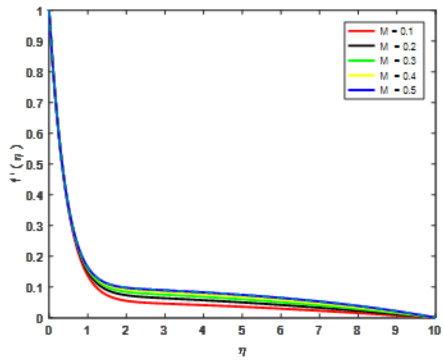

Figure 4: f' profile with the impact of M.

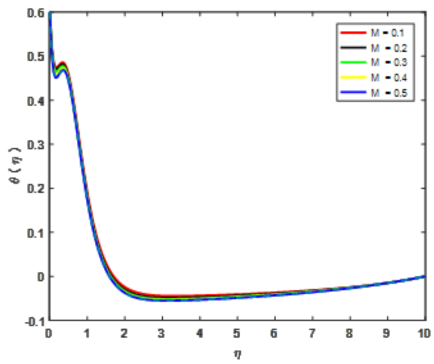

Figure 5: q profile with the impact of M.

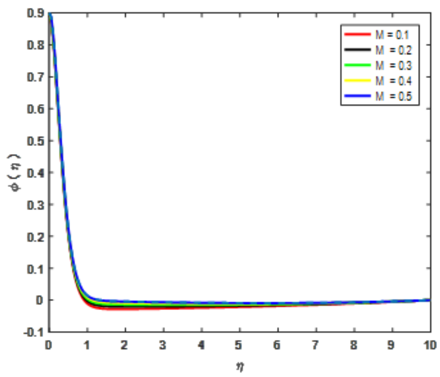

Figure 6: f profile through the impact of M.

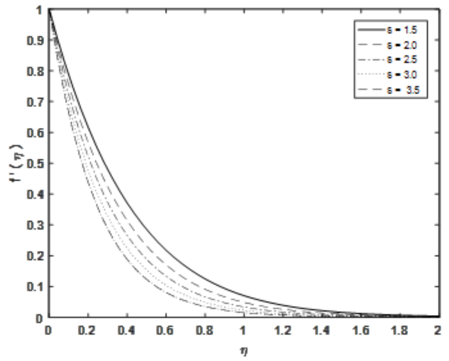

Figure 7: f' profile below the impact of s.

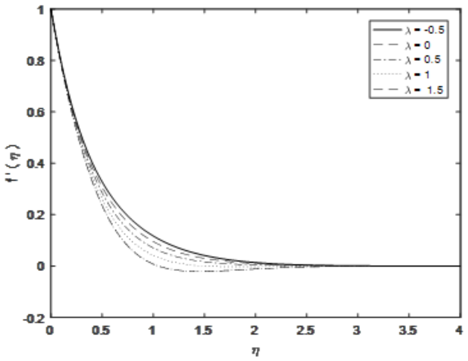

Figure 8: f' profile under the influence of λ.

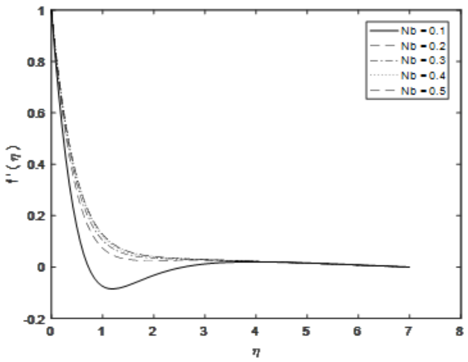

Figure 9: *f'* profile under the influence of N_b.

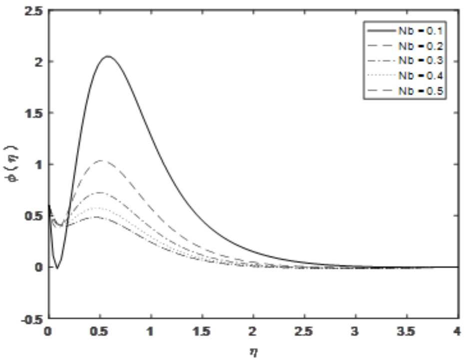

Figure 10: ϕ profile below the effect of N_b.

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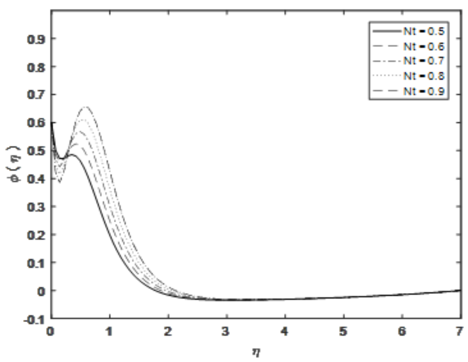

Figure 11: ϕ profile below the impact of N_t.

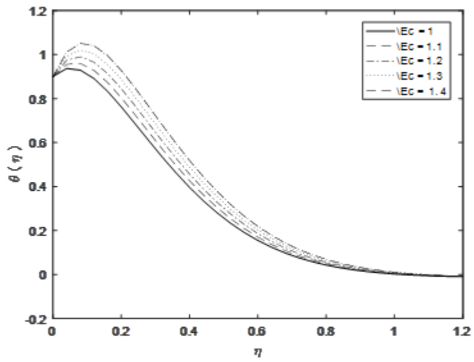

Figure 12: θ profile under the influence of Ec

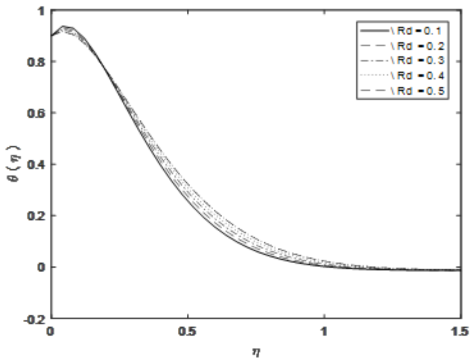

Figure 13: θ profile under the influence of R_d

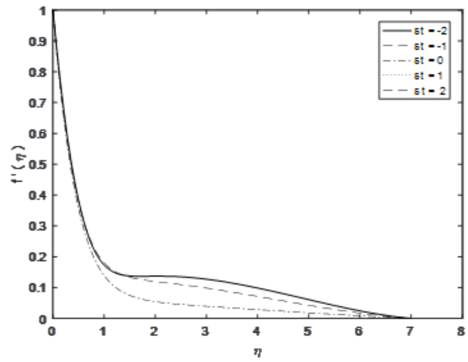

Figure 14: f' profile under the influence of S_t.

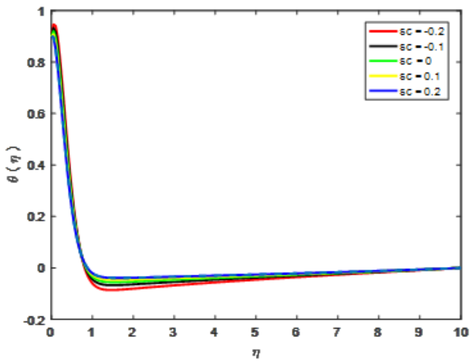

Figure 15: θ profile under the influence of S_c

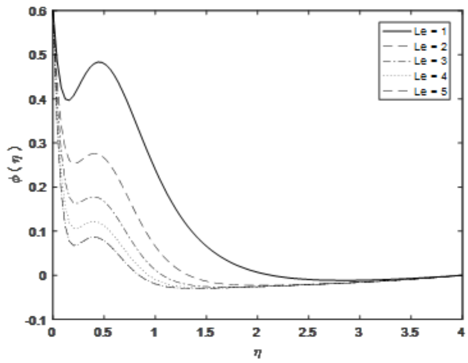

Figure 16: ϕ profile in the impact of Le.

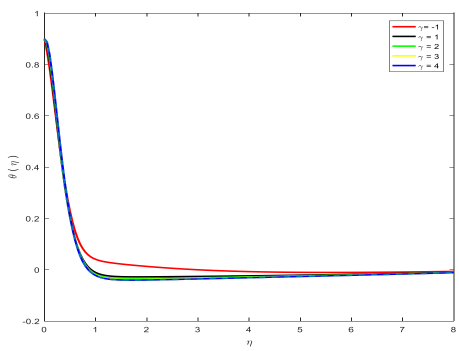

Figure 17: θ profile under the influence of γ.

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7. CONCLUSIONS

In this paper, combined properties of heat absorption/generation, thermal radiation with chemical reaction of unsteady mixed convection, MHD flow along with viscous dissipation and Joule heating of nanofluid over a stretchable sheet has been analysed numerically. Buongiorno model was utilized to study of impacts of the thermophoresis and Brownian motion.

The study has been deduced the following:

1. High electric parameter causes increase in temperature and velocity.
2. Viscous dissipation and thermal radiation increment the temperature profiles.
3. Heat sink decreases the temperature while reverse happened with heat source.
4. In case of generative chemical reaction, nanoparticle concentration decreases while an opposite trend may occur for the destructive chemical reaction.
5. Thermophoresis and Brownian motion parameters behaved in an opposite for the nanoparticles concentration.
6. Temperature profile decreases by thermal stratification whereas the nanoparticle concentration is reduced by concentration stratification.

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