SPECTRAL ANALYSIS MODEL OF SIMULATED N-TEAM INTERACTING DECISION MAKERS WITH BOUNDED RATIONALITY CONSTRAINTS

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ABSTRACT

The spectral analysis model was derived to forecast performance of interacting decision makers, based on the simulated mode that was adopted for the model of N-Teams of interacting decision maker (DM) with bounded rationality constraints. Using this model, the effectiveness (efficiency) of command input involving a feedback mechanism in a command, control, communication, and intelligence (C^3I) system in conjunction with decision support system (DSS) was investigated with a view to determining output (workload/performance of effectors/forecast) integrity. In a hierarchical organization, the performance of the last decision maker as was observed by Oladejo, was found to be greatly hampered by the colossal amount of bounded rationality constraints. The overall performance of the system was obtained using the Weibull distribution. The results here did not indicate obvious decline in the level of performance as expected when the process operates through the hierarchical structure. The overall performance which exhibited sinusoidal pattern was compared with the derived spectral analysis model Y, with the aid of the package number cruncher statistical system (NCSS). The trade-off between the derived optimal probability density function and the relative frequencies of forecasts was found to be insignificant this therefore establishes as the developed model acceptability. The goodness of fit of the spectral model of the original,(simulated deviates), corroborated this model acceptability with high values of coefficient multiple determination (R),low Fisher -F ratio in ANOVA, narrow confidence internal (CI), and insignificant autocorrelation factor (ACF)/portal autocorrelation factor (PACF).

Keywords: Decision Support, Simulation, decision maker, command and control, bounded rationality constraints, Computational Intelligence and Information Management Spectral analysis

INTRODUCTION

In earlier works done by Cooper and his colleagues (Cooper et al, 1986) and other authors (Cooper and Klein1980) and (Cooper et al 1979) on decision making at various stages were investigated and reported upon. Arising from the model of Levis and Boettcher (Levis and Boettcher, 1982) which couldn't address all the issues on command and control DM, led to generalization by Oladejo, (Oladejo,1995) who then developed hierarchical models of N-Teams of interacting DM with bounded rationality constraints that addressed a finite command structure. In them, the effectiveness or efficiency of command inputs were studied in relation to a command, control, communication and intelligence (C^3I) system, using DSS proposed by Hoil et al (Holt, et al, 1992). It was analytically shown that there was a colossal amount of bounded rationality constraints of order N^N imposed on the last (least or recipient), DM as contained in Oladejo (Oladejo, 1995). Embeded analysis involving the effort of Deckery (Deckery, 1984) Earl (Earl et al, 1981), Hitchins (Hitchins et al, 1989) and Johnson (Johnson et al, 1981) were integrated with simulation technique adopted from Holt (Holt et

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al1988) and Prister (Prister et al 1974), these were then employed to derive optimal models that were used to assess effectiveness of superior command input (or efficiency) and consequently, system output (workload, performance of effectors, or forecast).

The inputs were time signals which were assumed to be exponentially distributed, as reported by Feller (Feller, 1968). These were generated by random processed and coupled with DSS, in the form of situation report (SITREP) and were then fed into the system, as automated data. These input signals were processed in descending hierarchical order from superior to subordinate team members of the decision making process. Convolution strategies were then employed on the incoming signal to obtain reference signals. These reference signals were used as input to the next subordinate DM. The residual signal, not having binding effect on the superior DM, were re-injected as feedback to the next superior DM in order to optimize the decision making process, in line with Raven (Raven.5th Edition). On receiving the input signal and feedback, the DM concerned appraises the situation, and based on the available strategies with the aid of DSS, decides on the next line of action to be taken from a combination of tactics at his disposal in order to achieve the desired objective. This work briefly presents the simulation of the generalised model derived by Oladejo (Oladejo 1995). The adopted algorism for the implementation of this derived model was used by Oladejo and Ovuworie (Oladejo and Ovuworie, 2006), to asses adequacy of training models. the simulation output or the overall performance graphically exhibited sinusoidal pattern. This output was then used to develop the spectral analysis that could be used for forecasting the system performance more precisely.

This paper briefly, presents a simulation of a generalisation of the above in. Oladejo, (Oladejo, 1995). It also provides the algorithm, results, the spectral model, discussion and conclusion.

GENERAL REMARK

A vital statistic $d(Y,Y')$, which is the feedback (a by-product of output) from Intelligence Reports, Scout/Recce Patrols, and Physical-Electronic Surveillance, becomes part of input that furnishes the command and control with necessary requirements needed to effect changes that lead ultimately to the desired goal of each DM. This feed back in thence the choice of strategies to be taken. The chosen strategies are then convoluted to obtain command chices. These command choice were procured to obtain optimal output.

The High Command or the overall DM, receives the feed-backs from all other DMs. that are in the form of a convolution of exponentials with other distributions, which on further processing, produces Weibull distributed outputs, The particular C^3I model studied and a generalised feedback control system are shown in Figs 1 and 2 below, respectively. Both are typical of the Nigerian military apparatus.

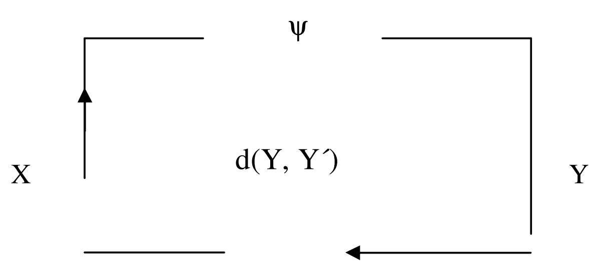

Figure 1. A simplification of the C^3I model

Adopted from Raven (Raven 5th Edition)

Embeded analyses involving Dockery (Dockery J 1984) Earl *et al* (Johnson JH 1981) Hitchins (Hitchins DK and Johnson 1989 (Johnson JH 1981),were integrated with simulation adaptation from Holt *et al* (Holt J, et al1988) and Prister (Raven 5th Edition)

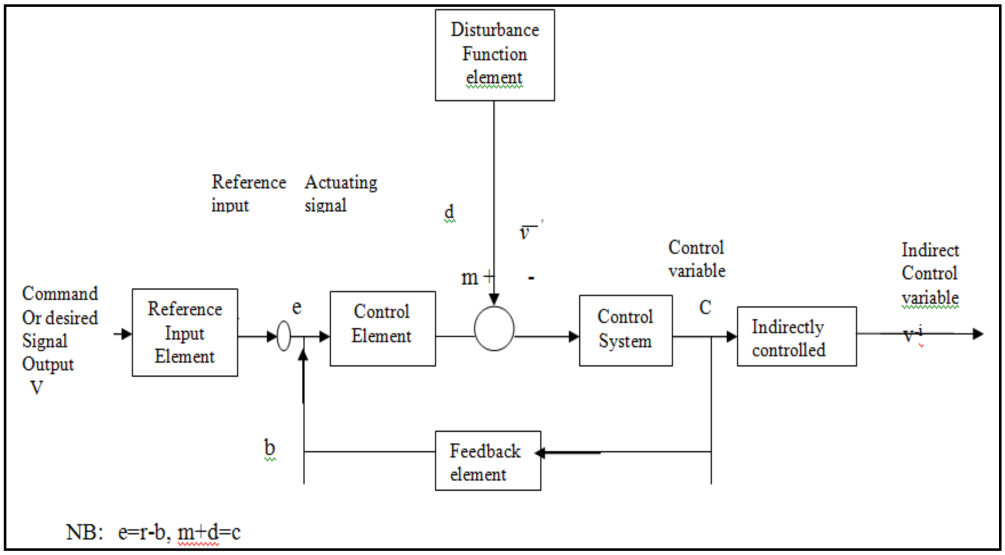

Figure 2. Generalised feedback control system by Raven [16]

Adopted from Raven (Raven 5th Edition)

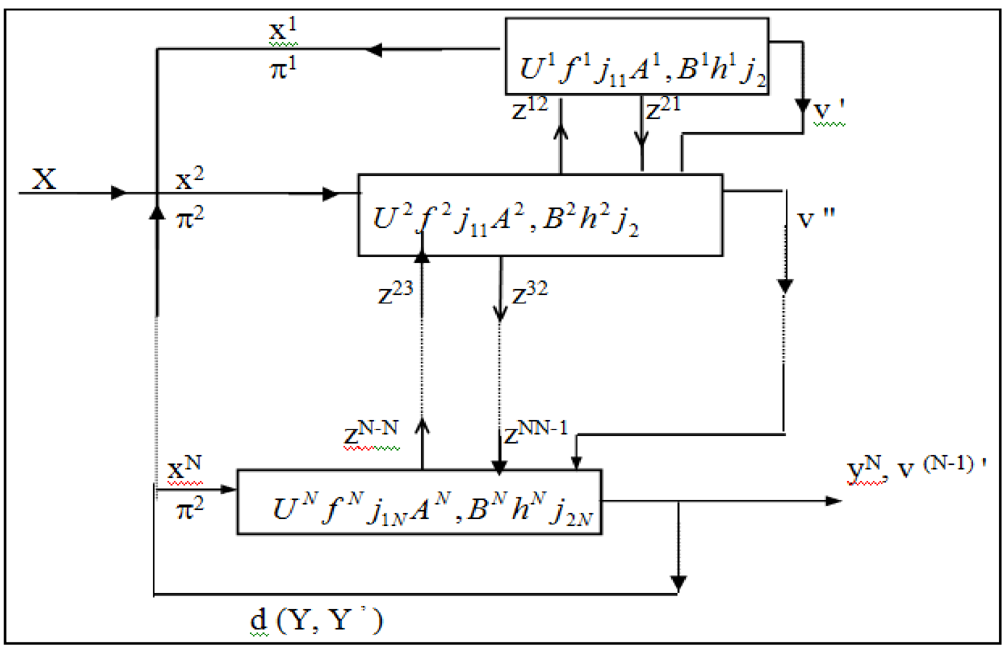

Figure 3. N-Team of Interacting Decision Makers with Bounded Rationality Constraints

Adopted from Oladejo (Oladejo, 1995).This is an extention of Fig. 2,by previous work of same author.

The Model: The model of N-Teams of interacting decision makers with bounded rationality constraints is decomposed into coherent component strategies which are then used to analyse the system of the specified C^3I.

Symbols

X^i is the input signal having exponential tempo/inter arrival times of the i-th team

π is portioning algorithm of input to respective DM

u is the internal decision

f^i is the algorithm for process u^i to obtain the battle scenario

q^i i-th team input regulator

z^i is the initial situational assessment.

$\bar{z}^i$ is the final situational assessment.

$g_C^{A^i}$, $g_C^{B^i}$ are the internal coordination strategies using SITREP z's and $\bar{v}^i$, the command Input to the i-th team

z' is situation assessment from other organisational members (SITREP or Feedback)

v is the final choice of decision.

v' is the command input

$\bar{v}^i$ is the output of DM i

$\bar{v}$ is final choice or response

Y is output or desired result

h_j is processing algorithm for final choice leading to Y.

h^i is the algorithm for process $\bar{v}^i$.

P(v | z) is response selection strategy that maps $\bar{z}$ to Y in the absence of v' and determines choice of h_j

P($\bar{v}$ | $\bar{z}$, v ') is response selection strategy that maps $\bar{z}$ and v' to v, it also determines choice of h_j

$H(x) = -\Sigma_x\, p(x) \log_2 p(x)$ is entropy of input, where p(x) is probability or uncertainty associated with N random variables, X

T(x, z', v': Y) is mutual information or transmission or throughput between inputs x, z', v' & output Y.

G_t is the throughput

G_b is the blockage

G_n is the noise

G_c is the internal coordination

|H| is the entropy of a random variable with propability p for taking one or two values, also which gives the amount of information transmitted.

A is the set of situational assessment options or strategies.

B is the set of available choices of tactics, modus operandi to remedy problem.

ψ is the convolution of strategies which is the assumed combined effects of process activities.

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Where

$j_{11} = 1,2,..U^1$, $j_{21} = 1,2,..V^1$, $j_{12} = 1,2,....U^2$, $j_{22} = 1,2,..V^2$,.........$j_{1N} = 1,2,....U^N$,....., $j_{2N} = 1,2,...V^N$

ANALYSIS

Strategies which are probability density functions (pdfs) are given as follows:

U^i (x^i) ~exponential inter arrival times or tempo of operations: $\frac{1}{\theta}e^{-t/\theta}$, t , $\theta > 0$

$f^i(u^i)$ ~normal because of a defined goal (i.e. goal is focus)

$$\frac{1}{\sqrt{(2\pi\sigma}} e^{-1/2[(x-\mu)^2/\sigma^2}$$, $-\infty < x, \mu < \infty$, $\sigma^2 > 0$

A (z^i) ~uniform distribution: $\frac{1}{N}$ or $\frac{x}{b-a}$, X = 0,1,2…..,N or a < X < b, respectively

v '~ triangular :

$$\frac{\overline{V}}{C}\ ,\ 0 < V \quad C\ , F = R = \frac{v^{-2}}{2C}\Big|_0^c\ , v^{-1} = 2R, (i.e \quad for \quad v^{-1} = c)$$

B ($\overline{z}$, $\overline{v}$| v´) ~ conditional jpdf of convoluted strategies which give:

g^k_C is internal coordination strategy of corresponding algorithm which depends on the distribution of their respective inputs.

$g_c^{A^1}$ ~ Bernoull: $p^x q^{1-x}$, x= 0, 1 (appropriate strategies are employed. Hence, $\sum x_i$ is binomial and all samples are appropriate. These are the internal coordination strategies among situational assessments represented by set A^1.)

$g_c^{B^1}$ ~ Geometric: pq^x, x= 1, 2... Depending on x (the strategies would result in a success hence $\sum x_1$ is negative binomial that all samples are successful. These are the internal coordination strategies among final choices and command inputs represented by set B^1.)

h(v´) ~ Weibull (due to reliability of subsystem before reaching final stage:
$\alpha\beta X^{\beta-1} e^{-\alpha}X^{\beta}$, X> 0 , α, β> 0)

μ (X) ~ expo (this is due to random occurrences)

F(u) ~ normal (i.e. geared towards a goal)

A(z) ~ uniform (simple random sample , equally likely samples)

B($\overline{z}$, $\overline{v}$| v´) ~conditional jpdf { (uniform x traingular) x (g^A_C g^B_C) yielding interaction due to z's to determine choices of v depending on v^{-1} from superior command.}

h($\overline{v}$) ~Weibull (output Y depends on survivability or reliability of other systems components).

The triangular pdf is due to the stringent nature of the command's input v '. The inverse of the transforms of the pdf to get negative deviates where obtained as follows:

R, RN1 are random numbers in the interval (0,1). The various distributions and random variable considered are:

1. $X = u^{-1}(x)$ is generated deviate

   $= \ln(1-R)/(-\alpha) = \ln R/\alpha$ since 1-R and R are all random numbers, $R \in$ RN1 and RN1 = (0,I) , α^{-1} is mean of the probability density function $\mu(x) = \alpha e^{-\alpha x}$

2. $f(x) \sim N(\mu,\sigma^{2)} = \frac{1}{\sqrt{(2\pi\sigma}} e^{-1/2!(x-\mu)^2/\sigma^2}$

   generated $x = \frac{1}{k} \ln\left(\frac{1+R}{1-R}\right)$, $k = \sqrt{\frac{8}{\pi}} =$ 1.595769122.

   or $x = \frac{\sigma}{\sqrt{2n/12}} \sum^{n}{}_{i=1} R_i + \frac{(\mu - n/2)}{\sqrt{2n/12}}$

   where $\sum R_1 \sim N(n/2, n/12)$ *and* $x \sim N(\mu,\sigma^2)$

3. (I) $f(x) = \frac{1}{N} : F(x) = \frac{x}{N}, x = 0,1 \ldots\ldots\ldots N$, generated x = NR

   (ii) $f(x) = \frac{1}{b-a} : F(x) = \frac{x}{b-a}$, a< x < b, generated x = (b-a)NR

4. $g^A{}_c$~bernoulli, generated x = 1 if R $\underline{<}$ p and zero otherwise and p is probability Of a success.

5. $g^B{}_c$~geometric, generated x = ln R/ln2, where p+q = 1

6. Binomial: generated $x = \sum^n_{i=1}\phi_I$; $R_I < p$, $\phi_I = 1$, and zero otherwise

7. Triangular: generated $\bar{v} = \sqrt{(2cR)} = 2R$, since c = 2

8. Negative Binomial ~ B (ꓕ, ꓦ| v´)

   i. $\frac{x}{2N(b-a)} (N)^2 p^N q^N$: generated deviate $X = 2NR^2 \sum\phi\sum\phi$

   ii. $\frac{1}{2(N-1)2} p^N q^N$: generated deviate $X = 2(b-a) R^2 \sum\phi\sum\phi$, for every $R, < \overline{P}$ $\phi_I = 1$

The generalised model as derived by Oladejo (13), is as shown below

<u>DM t (t = 1,2;....;N):</u>

$G^t = T(X^t, Z^{t(N-1)};\ldots\ldots;Z^{t1};V^{(N-1)1}Z^{1t};\ldots\ldots;Z^{(N-1)t};Y^t) + H(X^t; Z^{t(N-1)};$

U^t

$Z^{t1};V^{N-1)'}) - G^t{}_t + H(u^t) + H_z(v^t) + \Sigma_{i=1}[p_i g^i{}_c(p(X^t)) + \alpha_j$ ⵎ $(p_i)] +$

$H(Z^t, Z^{1t},\ldots\ldots,z^{(N-1)t}) + g^{At}{}_c(p(Z^t), p(Z^{t1}),\ldots\ldots,p(Z^{t(N-1)})) +$

V^t

$g^{Bt}_c (P(Z^t), p(V^{(N-1)'})) + \Sigma_{j=1} [p_j g^j_c(p(Z^t|V^t)) + \alpha_j |H| (p_j)] + H(Y^t) + H(Z^t)$
$+ H(Z^t) + H(Z^t; V^t) + T(X^t:Z^{t1};....;z^{t(N-1)}) + TZ^{-t}(X^t; Z^{t1};...;Z^{t(N-1)}:V^{(N-1)'})$

The generated deviates from a binomial distribution is $x = \sum^N \phi_I, \phi_I = 1$, if $R_i \leq p$ and p is the probability of the success of an event.

Derived Conditional Pdf

The conditional jpdf of bivariate distributed vector with some random variables that have binomial and negative binomial pairs, and some uniform and triangular pairs subject to command input $v^{/}$ given byis $B(\bar{z}, \bar{v} \mid v')$.

$$B(\bar{z}, \bar{v} \mid v') = 1/N\,(x/2)\,({}^{N}x)\,p^x q^{N-X}\,({}^{N-1}x)\,P^{N-X} q^x, \quad x = 0,1,\ldots, N$$
$$= 1/N\,(x/2)N\,({}^{N-1}x)\,p^x q^{N-X}\,({}^{N-1}x)\,p^{N-X} q^x$$
$$= ½\,({}^{N-1}x)^2 p^N q^N$$
$$= ½\,({}^{(N-1)}x)^2 p^N q^N \quad , \quad x = 0,1,\ldots, N$$

which is $\sim nb\,(2(N-1), 2x, p)$, a negative binomial pdf, when sampled deviates are from discrete uniform distribution.

$$B(\bar{z}, \bar{v} \mid v') = 1/(b-a)\;\; x/2\,({}^{N}x)\,p^x q^{N-X}\,({}^{N-1}x)\,p^{N-X} q^X \quad , \quad a \leq x \leq b,\; x = o,1,\ldots,N$$
$$= X/2(b-a)\,N\,({}^{N}x)^2\,p^N q^{N,}\;, \; x = a, a+1,\ldots\ldots, b\,(N), \text{ whichever is smaller.}$$

Where

$$x = \begin{cases} a, a+1,\ldots\ldots, b\;\; (b \leq N) \\ a, a+1,\ldots\ldots, N,\; (N \leq b) \end{cases}$$

when sampled deviates are from continuous uniform distribution.

These are continuous and discrete cases, respectively of negative binomial pdfs with parameters 2N and p.

9. $h(\bar{v})$~Weibull: generated $x = \left[-\alpha^{-1} \ln R\right]^{1/\beta}$

The chain of process is as follows:

U(x) → f(u) → A(z) → B(z , v/v ´) → h(v´) → v ‘= Y

Optimal Distributions

The optimal distributions are to ensure optimal outputs. The optimal distributions were derived thus:

Optimal Discrete (Situational Assessment) Distribution

$$B\left(\overline{Z},\overline{v} \mid v'\right)=\frac{1}{2}\binom{N-1}{x}^{2} p^{N}q^{N},\ x=0,1,2,\ldots,N,\ p+q=1.$$

$$\frac{d}{dp}\left(B\overline{Z},\overline{v} \mid v'\right)=\frac{N}{2}\binom{N-1}{x}^{2}\left[p^{N-1}q^{N}-p^{N}q^{N-1}\right]=0$$

$$\Rightarrow p^{N-1}q^{N}-p^{N}q^{N-1}=p^{N-1}q^{N-1}(q-p)=0\ \ or\ \ q=p$$

$$\therefore B^{*}\left(\overline{Z},\overline{v} \mid v'\right)=\frac{1}{2}\binom{N-1}{x}^{2} p^{2N},\ x=0,1,2,\ldots,N,\ the\ optimal\ \ pdf.$$

Optimal Continuous (Situational Assessment) Distribution

$$B\left(\overline{Z},\overline{v} \mid v'\right)=\frac{Nx}{2(b-a)}\binom{N}{x}^{2} p^{N}q^{N},\ x=a,a+1,\ldots,b\ (b\leq N)$$

$$=a,a+1,\ldots,N(N\leq b)\ and\ \ p+q=1$$

$$\frac{d}{dp}B^{*}\left(\overline{Z},\overline{v} \mid v'\right)=\frac{N^{2}x}{2(b-a)}\binom{N}{x}^{2}\left[p^{N-1}q^{N}-p^{N}q^{N-1}\right]=0$$

$$\Rightarrow p^{N-1}q^{N}-p^{N}q^{N-1}=p^{N-1}q^{N-1}(q-p)=0\ \ or\ \ q=p$$

$$\therefore B^{*}\left(\overline{Z},\overline{v} \mid v'\right)=\frac{Nx}{2(b-a)}\binom{N}{x}^{2} p^{2N},x=a,a+1,\ldots,b\ \ (b\leq N)$$

$$=a,a+1,\ldots,N(N\leq b),\ the\ optimal\ \ pdf.$$

METHODOLOGY

The implementation involved the following steps.

1. Generate random numbers R∈ (0, 1),
2. And use same to obtain random deviates from exponential pdf; u(x) = F_1 (x)
3. These exponentially generated random deviates in step 1 are now used to generate normal deviates for processing internal decision; $F_2(x)=f(\mu^{-1})$.
4. Obtain from an ordered scenario, deviates from the uniform pdf of the identified situated assessment, z, using results obtained in step 3 $F_3(x)=A(z)$
5. Use results in step 4 to generate tactical (ORBAT ≡ order of battle) deviates from the conditional pdf of bivariate distribution to get final choices, $\overline{v}=B(\overline{z},\overline{v} \mid v') \ = \ F_4(x)$
6. Use results in step 4 to generate Weibull variant output V^1 = Y from pdf $h(\overline{v})=F_5(x)$

The inverse transforms of the various strategies are then stated thus:

a. Random number: R

$$x=F_1^{-1}(\mu)=-\alpha^{-1}In\ \ R$$

b. Exponential:

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c. Normal: x = F_2^{-1} = 1/k ln $\left(\frac{1-R}{1-R}\right)$, k = √8/π = 1.595769122

d. Uniform (i) discrete: x = NR = F_3^{-1} ---- (i)

= (b – a)R ---- (ii)

e. Conditional pdf of bivariate distribution

I. discrete $F_4^{-1} = 2NR^2 \sum_1^{N-1}\phi_i$, where $\sum\Phi_I \ni \bar{v}$, $R_I \leq$ p, $\Phi_i = 1$

II. Continuous: $F_4^{-1} = 2(b\text{-}a)\ R^2 \sum^{N-1}\phi_i$, where $\sum\phi_i, \ni \bar{v}$, $R_I \leq p\Phi_i = 1$

f. Weilbull: X = F_5^{-1} = $\left(\frac{-\ln R}{\alpha}\right)^{1/\beta}$

This algorithm was developed from basic concepts in [16].

RESULTS

The results spectral model values using original data of the computed values are as shown on Table 1 below.

Table 1: Summary of the simulation of N-team interaction DM model

1. NB: 1. The use of I/F_i^{-1}, i=1,.....,5 was to obtain random deviates needed for generating next stage of the process because reciprocal is equally unique as the generated deviate, since values required are fractions.

2. From DM 2 to DM N, in col 6, R = F_3^{-1} or $1/F_3^{-1}$ if obtained value is > 1. Where F_5^{-1} is R = F_4^{-1} or 1/ F_4^{-1} in col 7 R = F_5^{-1} or ½ [$\bar{R}$ + F_5^{-1}]i.e. $\bar{R}$ from the predecessor of current DM.

   From the optimal pdfs for all values of p only small values of N yielded meaningful results, others were infinitesimal. It was also observed that as the p's increases the $B^*(\bar{Z}, \bar{v} \mid v')$ also increases, ranging between 0.0 and 1.1 (discrete) and 0.0 and 26 (continuous) which are unacceptable due to violation of probability law.

3. The values on the tables in the appendix were generated through the modification of the original results obtained using corresponding equations for each column in other to take care of missing and extreme values.

4. The results of the spectral analysis of the output on Table 2 are given on Tables 3 and 4.

5. Tables 5 and 6 give overall summary of the results for the discrete and continuous distributions respectively.

6. The forecasts do/don't show discrepancies with original data hence the derived model is acceptable.

Table 1. Summary of the simulation of N-team interaction DM model

			F3	F3=F3 if F3<1	280 (F3)	Modified	(-1n F4)1/1.5	
				F3 = 1/ F3 if F3>1	N = 15		(-lnF4) 0.66666666	MODIFIED
				F3	F4	F4	F5	F5
R	F1	F2	F3	MODIFIED F3	F4	MODIFIED F4	F5	MODIFIED F5
0.31	0.347377	0.454273	4.544273	0.2200572	13.559048	-0.0737515	1.8942328	0.5279182
0.14	0.196611	0.249667	2.496666	0.40054	44.921042	-0.0222613	2.437227	0.4103024
0.227	0.148281	0.187222	1.872224	0.53412	79.879569	-0.0125188	2.6772183	0.373522
0.221	0.150959	0.190657	1.906566	0.524503	77.028951	-0.0129821	2.6623918	0.3756021
0.088	0.243042	0.310827	3.108274	0.321722	28.981413	-0.0345049	2.2462894	0.4451786
0.024	0.37297	0.491135	4.911346	0.20361	11.607969	-0.0861477	1.8182095	0.5499916
0.562	0.057625	0.072303	0.723028	0.723028	146.37546	-0.0068317	2.9186253	0.3426
0.944	0.005763	0.007222	0.072228	0.072228	1.4607275	-0.6845904	0.5236521	0.5236521
0.73	0.031471	0.039456	0.394562	0.394562	43.590168	-0.022941	2.4243671	0.4124788
0.81	0.021072	0.026414	0.264139	0.265139	19.535435	-0.5118903	0.765414	0.765414
3.63	0.045728	0.057352	0.573522	0.573522	92.099696	-0.0108578	2.7349088	0.3656429
0.564	0.05727	0.071856	0.718561	0.718561	144.57238	0.006917	2.9137866	0.343196
0.972	0.00284	0.035594	0.05339	0.05339	0.7981378	0.7981378	0.3704512	0.3704512
0.376	0.097817	0.122988	1.227882	0.814411	185.71428	0.0053846	3.0107895	0.3321388
0.663	0.041098	0.051538	0.5153777	0.5153777	74.371966	0.0134459	2.6480305	0.3776392
0.26	0.134707	0.090669	0.906695	0.906695	230.18683	0.0043443	3.0927175	0.3233402
0.987	0.001309	0.00164	0.0163999	0.0163999	0.075308	0.075308	1.884103	0.5307566
0.705	0.034956	0.043828	0.4382839	0.4382839	53.785975	0.0185922	2.5135439	0.3978447
0.234	0.145243	0.183332	1.833321	0.545458	83.30684	0.0120038	2.694308	0.3711528
0.384	0.095711	0.120325	1.203246	0.831085	193.40757	0.0051704	3.0263645	0.3304295
0.847	0.016605	0.020814	0.208138	0.208138	12.13	0.0824402	1.839894	0.5435096
0.497	0.069917	0.087771	0.877706	0.877706	215.70299	0.04636	3.0680318	0.3259419
0.448	0.080796	0.100853	0.008535	0.991537	275.28077	0.0036327	3.1601698	0.3164387
0.321	0.113631	0.143034	1.430336	0.699136	136.86152	0.0073067	2.8923406	0.3457407
0.401	0.091379	0.114847	1.148475	0.87072	212.28293	0.0047107	3.0619457	0.3265897
0.227	0.148281	0.187222	1.872222	0.534125	79.881064	0.0125186	2.6772259	0.373521

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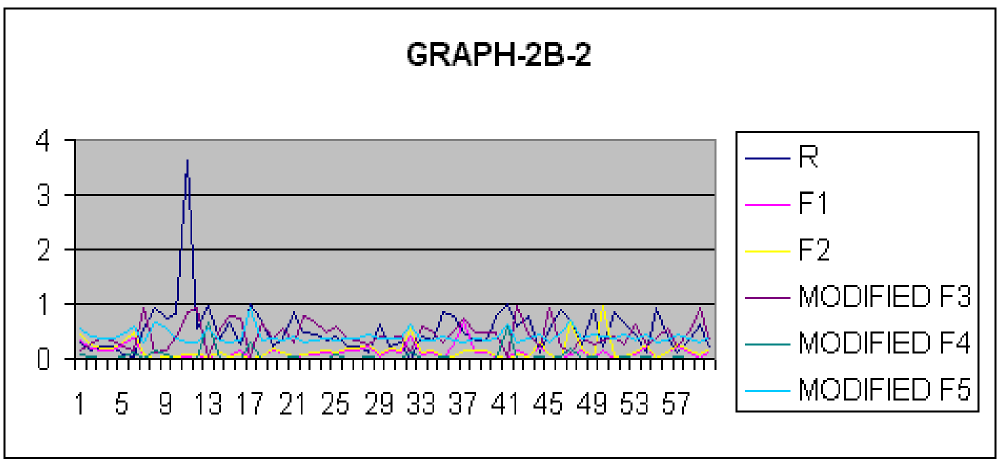
0.222	0.150508	0.190078	1.900776	0.526101	77.499033	0.0129034	2.664877	0.3752518
0.113	0.218037	0.277727	2.777268	0.360066	36.301307	0.0275472	2.3453758	0.4263709
0.619	0.047965	0.060161	0.611614	0.611614	104.74007	0.0095474	2.7865118	0.3588716
0.238	0.143548	0.181163	1.81163	0.551989	85.31372	0.0117214	2.7039675	0.3698269
0.283	0.126223	0.159055	1.590553	0.628712	110.67806	0.0090352	2.8084914	0.356063

NB: 1.The use of I/F_i^{-1},i=1,.....,5 was to obtain random deviates needed for generating next stage of the process because reciprocal is equally unique as the generated deviate, since values required are fractions.

2. From DM 2 to DM N, in col 6, $R = F_3^{-1}$ or $1/F_3^{-1}$ if obtained value is > 1.where F_5^{-1} is $R = F_4^{-1}$ or $1/ F_4^{-1}$ in col 7 $R = F_5^{-1}$ or ½ [$\overline{R} + F_5^{-1}$]i.e. $\overline{R}$ from the predecessor of current DM.

2. The values on the tables in the appendix were generated through the modification of the original results obtained using corresponding equations for each column in other to take care of missing and extreme values.

3. The columns containing modified values were obtained by the coputations: In column 5 F3 = F3 if F3 < 1 otherwise F3 = 1 / F3 if F3> 1; column 6 F4 = 280F3, N = 15; column 7 F4 = F4 if F4 < 1 otherwise F4 = 1 /F4 if F4 > 1; column 8 $F5 = (-\ln F4)^{1.5} (-\ln F4)^{0.6666}$ and column 9 F5 = F5 if F5 < 1 otherwise F5 = 1/F5 if F5 > 1.



Graph 1. Graph of deviates from original and modified results

SPECTRAL MODEL

The values in Modified F5, the overall output were fed into NCSS package and allowed to run several iterations. The tentative model was assumed and their parameters were estimated. The test was conducted for their significance. The results are as shown on Table 3 and 4 below. The choice of spectral analysis to derive the model for this research was predicated on the sinusoidal outputs, as depicted on graphs of original data.

Table 2. Fourier Analysis of F5 (0,0,12,1,0) Discrete

Freq	*Wavelength*	*Period*	*Cosine(a's)*	*Sine(b's)*	*I*	*RSE*	*Spectrum (y)*	*T*
0.2010619	31.25	1.306727	0.1557562	0.7109456	3.9727773	0.248298581	1.081139	*1.594148941*
0.2764601	22.72727	0.319731	-0.2288242	0.2779348	0.972062007	0.060753875	0.5704369	-0.26550459
0.3518584	17.85714	8.49E-02	-7.27E-02	0.1706389	0.257974454	0.016123403	0.241798	-0.221912898
0.4272566	14.70588	0.3208101	-0.2979069	0.2032172	0.975343136	0.060958946	0.330327	-0.204624149
0.5026549	12.5	0.585318	-0.4463921	-0.1949425	1.779513639	0.111219602	0.5963239	-0.249152264
0.5780531	10.86957	0.8828436	-0.1544788	-0.5779375	2.684065902	0.167754119	0.8930968	-1.085936186
0.6534513	9.615385	1.211129	0.3859549	-0.5847993	3.682135546	0.230133472	1.068159	*0.596270372*
0.7288495	8.620689	1.110506	0.6676783	-6.61E-02	3.37621566	0.211013479	0.9247369	0.316040642
0.8042477	7.8125	0.452576	0.3128557	0.2925417	1.375945014	0.085996563	0.5476763	0.274876128
0.8796459	7.142857	7.99E-02	0.1073962	0.1444787	0.243060289	0.015191268	0.231044	0.141450704
0.9550442	6.578948	0.1606085	8.89E-02	0.2391666	0.488289993	0.030518125	0.1837093	0.343254291
1.030442	6.097561	0.310572	-0.3231932	0.1464299	0.944216701	0.059013544	0.3933354	-0.182595252
1.105841	5.681818	0.7088259	-0.2681032	-0.4641715	2.155008804	0.13468805	0.6428744	-0.5023739
1.181239	5.319149	0.9092254	0.322379	-0.5144333	2.764273798	0.172767112	0.7141021	*0.535913048*
1.256637	5	0.5242552	0.436524	-0.1481973	1.593867317	0.099616707	0.576987	0.228204423

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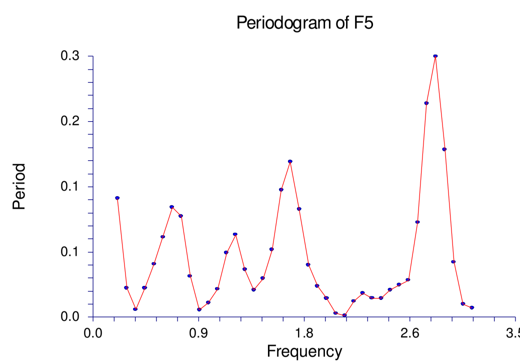
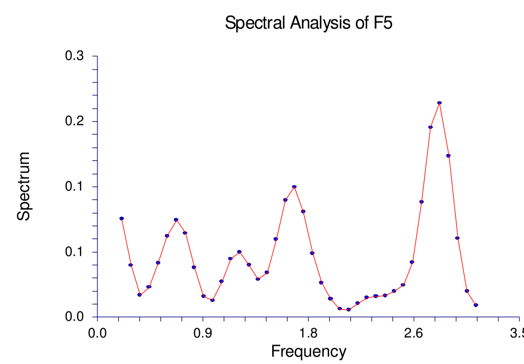
1.332035	4.716981	0.2974802	0.3345454	-0.093102	0.904414553	0.05652591	0.4163755	0.168963344
1.407434	4.464286	0.427391	0.4041789	-9.94E-02	1.299375736	0.081210983	0.4899177	0.20092831
1.482832	4.237288	0.7448819	0.5493824	-1.14E-02	2.264628282	0.141539268	0.8569692	0.257633422
1.55823	4.032258	1.398635	0.6623496	0.3581244	4.252200589	0.265762537	1.285192	0.401242088
1.633628	3.846154	1.712059	0.2053125	0.8073777	5.205089798	0.325318112	1.433074	*1.584502222*
1.709026	3.676471	1.188529	-0.4980477	0.4834653	3.613426558	0.22583916	1.158505	-0.453448856
1.784425	3.521127	0.5749256	-0.4171064	-0.2430596	1.747917886	0.109244868	0.7020625	-0.261911272
1.859823	3.378378	0.3427328	0.1398483	-0.3455068	1.041993719	0.065124607	0.3759642	0.465680365
1.935221	3.246753	0.2102343	0.2919239	1.56E-03	0.63916491	0.039947807	0.1987748	0.136843221
2.010619	3.125	4.34E-02	7.76E-02	0.1074657	0.131817625	0.008238602	9.08E-02	0.106123086
2.086018	3.012048	1.89E-02	4.80E-02	-7.31E-02	0.057330394	0.00358315	7.96E-02	0.074604312
2.161416	2.906977	0.1766833	0.2650948	-3.67E-02	0.537161586	0.033572599	0.1530112	0.126643748
2.236814	2.808989	0.2634931	0.2067052	0.2531488	0.80108516	0.050067822	0.2168496	0.242218495
2.312212	2.717391	0.2103724	-0.1422822	0.2550172	0.639584975	0.039974061	0.2270195	-0.280949135
2.38761	2.631579	0.2071929	-0.2820388	-6.67E-02	0.629918273	0.039369892	0.2383885	-0.13959034
2.463009	2.55102	0.2976003	-0.1115786	-0.3289187	0.904779714	0.056548732	0.2863978	-0.506806252
2.538407	2.475248	0.3544004	0.1251373	-0.3577748	1.077466135	0.067341633	0.3535165	*0.538141972*

2.613805	2.403846	0.4085487	0.226265	-0.3382547	1.242090692	0.077630668	0.6020741	0.343096229
2.689203	2.336449	1.043273	0.4901293	-0.4274127	3.171812601	0.198238288	1.267862	0.404461206
2.764601	2.272727	2.351764	0.9758813	-3.13E-02	7.149952919	0.446872057	2.089236	0.457916406
2.84	2.212389	2.872672	0.7174597	0.8060628	8.73364244	0.545852653	2.35686	*0.76081298*
2.915398	2.155172	1.846145	-0.1677988	0.8486516	5.612744816	0.350796551	1.77555	-2.090578425
2.990796	2.10084	0.6078328	-0.4239714	0.258154	1.847964268	0.115497767	0.8667505	-0.272418769
3.066195	2.04918	0.1462731	-0.2408232	3.60E-02	0.444706798	0.027794175	0.2860245	-0.115413195
3.141593	2	0.1039675	-0.2052924	1.16E-15	0.316087271	0.019755454	0.1321712	-0.096230813





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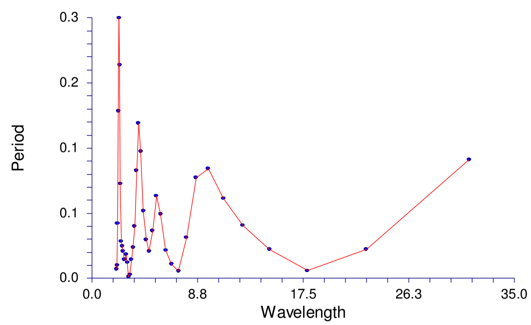

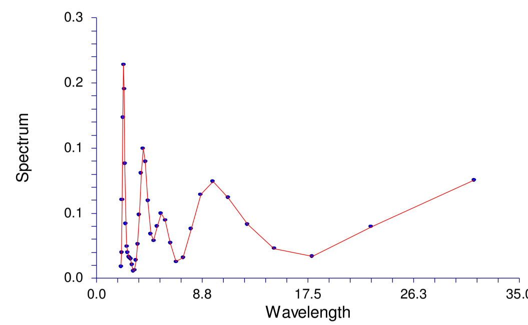

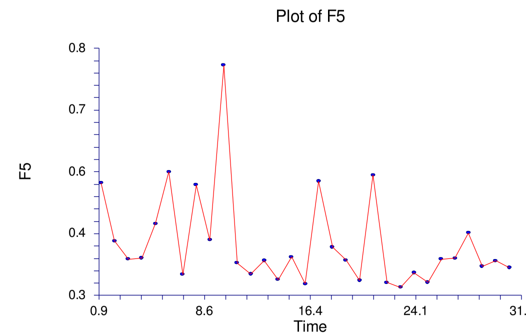

Data Plot Section

Graph 2. Fourier plots of discrete pdf

Table 3. Fourier Analysis of R (0,0,12,1,0) continuous

Frequency	*Wavelength*	*Period*	*Cosine(a's)*	*Sine(b's)*	*I*	*RSE*	*Spectrum (y)*	*T*
0.2010619	31.25	92.51227	-4.455503	4.201178	281.2605268	17.57878292	73.43137	-3.945409289
0.2764601	22.72727	92.33107	-6.008712	-1.150363	280.709662	17.54435387	78.02203	-2.919819401
0.3518584	17.85714	49.22277	-3.303104	-3.007122	149.6495907	9.353099418	64.5005	-2.831609122
0.4272566	14.70588	51.94764	-2.744674	-3.677582	157.9338355	9.870864719	59.11039	-3.596370541
0.5026549	12.5	76.16074	-0.2083686	-5.552443	231.5478056	14.47173785	63.58086	-69.45258474
0.5780531	10.86957	62.63419	3.459103	-3.663936	190.4236543	11.9014784	57.43106	*3.440625617*
0.6534513	9.615385	33.49823	3.675931	-0.2580795	101.8430531	6.365190818	38.42026	*1.731586044*

0.7288495	8.620689	19.12835	2.302111	1.566614	58.15495861	3.634684913	22.60828	*1.57884868*
0.8042477	7.8125	15.19826	0.4792288	2.435408	46.20654277	2.887908923	16.30645	*6.026158952*
0.8796459	7.142857	14.59274	-1.47216	1.936016	44.36559763	2.772849852	15.78318	-1.883524788
0.9550442	6.578948	17.55852	-2.643092	0.362914	53.38231419	3.336394637	18.58836	-1.262307418
1.030442	6.097561	23.61381	-2.366753	-1.99267	71.79190119	4.486993824	21.99888	-1.895843725
1.105841	5.681818	24.82432	0.254272	-3.162009	75.47216375	4.717010234	22.12983	*18.55104075*
1.181239	5.319149	17.95136	2.374001	-1.281018	54.57665898	3.411041186	18.10925	*1.436832245*
1.256637	5	11.55208	1.966705	0.902719	35.12122612	2.195076633	13.6842	*1.116118906*
1.332035	4.716981	11.54916	0.6103806	2.075833	35.11235341	2.194522088	12.72307	*3.595333941*
1.407434	4.464286	15.06796	-1.499165	1.96483	45.8103947	2.863149668	12.98331	-1.909829584
1.482832	4.237288	12.3328	-2.235895	9.05E-03	37.49481272	2.343425795	10.70346	-1.048092954
1.55823	4.032258	4.709627	-1.306734	-0.4489679	14.31844441	0.894902776	10.454	-0.684839283
1.633628	3.846154	14.31956	-2.340561	-0.5713554	43.53504591	2.720940369	19.75043	-1.162516324
1.709026	3.676471	40.2221	-2.054682	-3.476058	122.2852301	7.642826878	34.69803	-3.719712772
1.784425	3.521127	49.55243	1.751606	-4.125383	150.6518136	9.415738348	42.37623	*5.37548875*
1.859823	3.378378	37.35416	3.679075	-1.267496	113.5660422	7.09787764	38.85266	*1.929256033*
1.935221	3.246753	29.65139	3.264007	1.168737	90.14765903	5.63422869	33.3431	*1.726169303*

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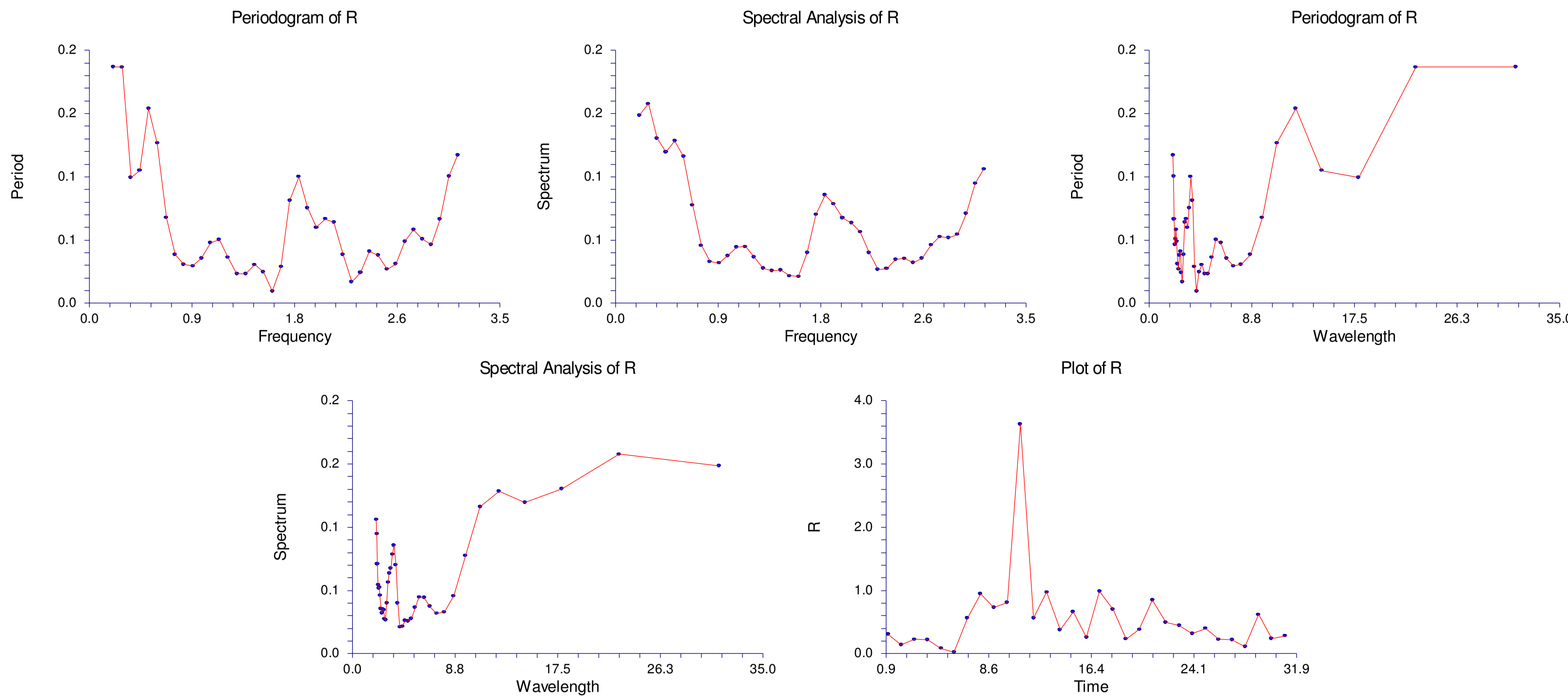
2.010619	3.125	33.02376	1.586286	3.297034	100.4005235	6.275032721	31.48339	*3.955801615*
2.086018	3.012048	31.77502	-1.776333	3.118522	96.60403794	6.037752371	27.96455	-3.398998032
2.161416	2.906977	19.09488	-2.76453	0.3127378	58.05323289	3.628327056	19.75231	-1.312457111
2.236814	2.808989	8.387024	-1.694878	-0.7260909	25.49864572	1.593665358	13.1924	-0.940283229
2.312212	2.717391	12.09528	-1.713938	-1.401943	36.77270732	2.298294208	13.5847	-1.340943609
2.38761	2.631579	20.27179	-0.2759208	-2.85331	61.63132683	3.851957927	17.08503	-13.96037532
2.463009	2.55102	18.888	1.986715	-1.926015	57.42427704	3.589017315	17.50347	*1.806508389*
2.538407	2.475248	13.35063	2.322663	-0.1309131	40.58926238	2.536828899	15.89655	*1.092207048*
2.613805	2.403846	15.45102	2.164267	1.256695	46.97500478	2.935937799	17.64941	*1.356550647*
2.689203	2.336449	24.14657	0.7058266	3.047954	73.41161082	4.588225676	22.79594	*6.500499805*
2.764601	2.272727	28.79023	-2.236553	2.582331	87.52952037	5.470595023	26.01021	-2.445993912
2.84	2.212389	25.09383	-3.189364	1.28E-02	76.29154839	4.768221774	25.60515	-1.495038438
2.915398	2.155172	22.9314	-2.448658	-1.816508	69.71720486	4.357325304	26.95971	-1.77947484
2.990796	2.10084	32.85389	-1.011626	-3.50635	99.88408115	6.242755072	35.16671	-6.171010899
3.066195	2.04918	49.71485	2.482546	-3.740283	151.1456367	9.446602295	46.85827	*3.805207354*
3.141593	2	58.00608	4.849097	-1.89E-13	176.3530629	11.02206643	52.47859	*2.273014219*



Data Plot Section

Graph 3. Fourier plots of continuous pdf

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Table 4. Summary of Results for Discrete Distribution

R	X = F1	BD	BC	F5	B*	Y	P(Y)	YB trade-offs	Remark
0.31	0.34738	5.00E-11	0	0.2010619	5.00E-11	1.081139	0.00300752	-0.00301	
0.14	0.19661	2.00E-10	6.25E-10	0.2764601	6.25E-10	0.5704369	0.00413534	-0.00414	
0.227	0.14828	3.00E-10	5.00E-09	0.3518584	5.00E-09	0.241798	0.00526316	-0.00526	
0.221	0.15096	2.00E-10	7.50E-09	0.4272566	7.50E-09	0.330327	0.00639098	-0.00639	
0.088	0.24304	5.00E-11	2.50E-09	0.5026549	2.50E-09	0.5963239	0.0075188	-0.00752	
0.024	0.37297	5.12E-08	1.25E-10	0.5780531	5.12E-08	0.8930968	0.00864662	-0.00865	
0.562	0.05763	2.05E-07	0	0.6534513	2.05E-07	1.068159	0.00977444	-0.00977	
0.944	0.00576	3.07E-07	6.40E-07	0.7288495	6.40E-07	0.9247369	0.01090226	-0.0109	
0.73	0.03147	2.05E-07	5.12E-06	0.8042477	5.12E-06	0.5476763	0.01203007	-0.01202	
0.81	0.02107	5.12E-08	7.68E-06	0.8796459	7.68E-06	0.231044	0.01315789	-0.01315	
3.63	0.04573	2.95E-06	2.56E-06	0.9550442	2.95E-06	0.1837093	0.01428571	-0.01428	
0.564	0.05727	1.18E-05	1.28E-07	1.030442	1.18E-05	0.3933354	0.01541353	-0.0154	
0.972	0.00284	1.77E-05	0	1.105841	1.77E-05	0.6428744	0.01654136	-0.01652	
0.376	0.09782	1.18E-05	3.69E-05	1.181239	3.69E-05	0.7141021	0.01766918	-0.01763	
0.663	0.0411	2.95E-06	2.95E-04	1.256637	2.95E-04	0.576987	0.01879699	-0.0185	

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0.26	0.13471	5.24E-05	4.43E-04	1.332035	4.43E-04	0.4163755	0.01992481	-0.01948
0.987	0.00131	2.10E-04	1.48E-04	1.407434	2.10E-04	0.4899177	0.02105264	-0.02084
0.705	0.03496	3.15E-04	7.38E-06	1.482832	3.15E-04	0.8569692	0.02218045	-0.02187
0.234	0.14524	2.10E-04	0	1.55823	2.10E-04	1.285192	0.02330827	-0.0231
0.384	0.09571	5.24E-05	6.55E-04	1.633628	6.55E-04	1.433074	0.02443609	-0.02378
0.847	0.01661	4.88E-04	0.0052	1.709026	5.20E-03	1.158505	0.0255639	-0.02036
0.497	0.06992	0.002	0.0079	1.784425	7.90E-03	0.7020625	0.02669173	-0.01879
0.448	0.0808	0.0029	0.0026	1.859823	2.90E-03	0.3759642	0.02781955	-0.02492
0.321	0.11363	0.002	1.31E-04	1.935221	2.00E-03	0.1987748	0.02894737	-0.02695
0.401	0.09138	4.88E-04	0	2.010619	4.88E-04	9.08E-02	0.03007518	-0.02959
0.227	0.14828	0.003	0.0061	2.086018	6.10E-03	7.96E-02	0.03120301	-0.0251
0.222	0.15051	0.0121	0.0488	2.161416	4.88E-02	0.1530112	0.03233083	0.016469
0.113	0.21804	0.0181	0.0732	2.236814	7.32E-02	0.2168496	0.03345865	0.039741
0.619	0.04797	0.0121	0.0244	2.312212	2.44E-02	0.2270195	0.03458646	-0.01019
0.238	0.14355	0.003	0.0012	2.38761	3.00E-03	0.2383885	0.03571428	-0.03271
0.283	0.12622	0.0141	0	2.463009	1.41E-02	0.2863978	0.03684211	-0.02274
		0.0565	0.0378	2.538407	5.65E-02	0.3535165	0.03796993	0.01853

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0.0847	0.3023	2.613805	3.02E-01	0.6020741	0.03909774	0.263202
0.0565	0.4535	2.689203	4.54E-01	1.267862	0.04022556	0.413274
0.0141	0.1512	2.764601	1.51E-01	2.089236	0.04135337	0.109847
0.0537	0.0076	2.84	5.37E-02	2.35686	0.04248121	0.011219
0.2147	0	2.915398	2.15E-01	1.77555	0.04360902	0.171091
0.3221	0.1765	2.990796	3.22E-01	0.8667505	0.04473684	0.277363
0.2147	1.4124	3.066195	1.41E+00	0.2860245	0.04586467	1.366535
0.0537	2.1186	3.141593	2.12E+00	0.1321712	0.04699249	2.071608
0.1743	0.7062		7.06E-01			0.7062
0.6974	0.0353		6.97E-01			0.6974
1.046	0		1.05E+00			1.046
0.6974	0.6711		6.97E-01			0.6974
0.1743	5.3687		5.37E+00			5.3687
	8.0531		8.05E+00			8.0531
	2.6844		2.68E+00			2.6844
	0.1342		1.34E-01			0.1342
	0		0.00E+00			0

2.1792	2.18E+00	2.1792
17.4339	1.74E+01	17.4339
26.1509	2.62E+01	26.1509
8.717	8.72E+00	8.717
0.4358	4.36E-01	0.4358

The Fourier model derived from results on Table 2 is given by

$Y^{`} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$= 0.1557562 cos θ - 0.7109456 sinθ + 0.3859549 cos2θ - 0.5847993 sin2θ + 0.322379 cos3θ - 0.5144333 sin3θ + 0.2053125 cos 4θ +0.8073777 sin 4θ + 0.1251373 cos 5θ - 0.3577748 sin 5θ +0.7174597 cos 6θ +0.8060628 sin 6θ

Table 5. Forecasts by Derived Fourier Model (θ= 12^0) Discrete

Y= F5	*Y'*
0.2010619	-0.25004
0.6534513	0.693296
1.181239	0.468951
1.633628	-0.7517
2.538407	-0.01013
2.84	-0.48937

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Table 6. Summary of Results for Continuous Distribution

R	*X = F1*	*BD*	*BC*	*F5*	B^*	*Y*	*P(Y)*	*YB trade-offs*	*Remark*
0.31	0.34738	5.00E-11	0	0.2010619	5.00E-11	1.081139	0.00300752	-0.00301	
0.14	0.19661	2.00E-10	6.25E-10	0.2764601	6.25E-10	0.5704369	0.00413534	-0.00414	
0.227	0.14828	3.00E-10	5.00E-09	0.3518584	5.00E-09	0.241798	0.00526316	-0.00526	
0.221	0.15096	2.00E-10	7.50E-09	0.4272566	7.50E-09	0.330327	0.00639098	-0.00639	
0.088	0.24304	5.00E-11	2.50E-09	0.5026549	2.50E-09	0.5963239	0.0075188	-0.00752	
0.024	0.37297	5.12E-08	1.25E-10	0.5780531	5.12E-08	0.8930968	0.00864662	-0.00865	
0.562	0.05763	2.05E-07	0	0.6534513	2.05E-07	1.068159	0.00977444	-0.00977	
0.944	0.00576	3.07E-07	6.40E-07	0.7288495	6.40E-07	0.9247369	0.01090226	-0.0109	
0.73	0.03147	2.05E-07	5.12E-06	0.8042477	5.12E-06	0.5476763	0.01203007	-0.01202	
0.81	0.02107	5.12E-08	7.68E-06	0.8796459	7.68E-06	0.231044	0.01315789	-0.01315	
3.63	0.04573	2.95E-06	2.56E-06	0.9550442	2.95E-06	0.1837093	0.01428571	-0.01428	
0.564	0.05727	1.18E-05	1.28E-07	1.030442	1.18E-05	0.3933354	0.01541353	-0.0154	
0.972	0.00284	1.77E-05	0	1.105841	1.77E-05	0.6428744	0.01654136	-0.01652	
0.376	0.09782	1.18E-05	3.69E-05	1.181239	3.69E-05	0.7141021	0.01766918	-0.01763	
0.663	0.0411	2.95E-06	2.95E-04	1.256637	2.95E-04	0.576987	0.01879699	-0.0185	

0.26	0.13471	5.24E-05	4.43E-04	1.332035	4.43E-04	0.4163755	0.01992481	-0.01948
0.987	0.00131	2.10E-04	1.48E-04	1.407434	2.10E-04	0.4899177	0.02105264	-0.02084
0.705	0.03496	3.15E-04	7.38E-06	1.482832	3.15E-04	0.8569692	0.02218045	-0.02187
0.234	0.14524	2.10E-04	0	1.55823	2.10E-04	1.285192	0.02330827	-0.0231
0.384	0.09571	5.24E-05	6.55E-04	1.633628	6.55E-04	1.433074	0.02443609	-0.02378
0.847	0.01661	4.88E-04	0.0052	1.709026	5.20E-03	1.158505	0.0255639	-0.02036
0.497	0.06992	0.002	0.0079	1.784425	7.90E-03	0.7020625	0.02669173	-0.01879
0.448	0.0808	0.0029	0.0026	1.859823	2.90E-03	0.3759642	0.02781955	-0.02492
0.321	0.11363	0.002	1.31E-04	1.935221	2.00E-03	0.1987748	0.02894737	-0.02695
0.401	0.09138	4.88E-04	0	2.010619	4.88E-04	9.08E-02	0.03007518	-0.02959
0.227	0.14828	0.003	0.0061	2.086018	6.10E-03	7.96E-02	0.03120301	-0.0251
0.222	0.15051	0.0121	0.0488	2.161416	4.88E-02	0.1530112	0.03233083	0.016469
0.113	0.21804	0.0181	0.0732	2.236814	7.32E-02	0.2168496	0.03345865	0.039741
0.619	0.04797	0.0121	0.0244	2.312212	2.44E-02	0.2270195	0.03458646	-0.01019
0.238	0.14355	0.003	0.0012	2.38761	3.00E-03	0.2383885	0.03571428	-0.03271
0.283	0.12622	0.0141	0	2.463009	1.41E-02	0.2863978	0.03684211	-0.02274
		0.0565	0.0378	2.538407	5.65E-02	0.3535165	0.03796993	0.01853

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0.0141	0.1512	2.764601	1.51E-01	2.089236	0.04135337	0.109847
0.0537	0.0076	2.84	5.37E-02	2.35686	0.04248121	0.011219
0.2147	0	2.915398	2.15E-01	1.77555	0.04360902	0.171091
0.3221	0.1765	2.990796	3.22E-01	0.8667505	0.04473684	0.277363
0.2147	1.4124	3.066195	1.41E+00	0.2860245	0.04586467	1.366535
0.0537	2.1186	3.141593	2.12E+00	0.1321712	0.04699249	2.071608
0.1743	0.7062		7.06E-01			0.7062
0.6974	0.0353		6.97E-01			0.6974
1.046	0		1.05E+00			1.046
0.6974	0.6711		6.97E-01			0.6974
0.1743	5.3687		5.37E+00			5.3687
	8.0531		8.05E+00			8.0531
	2.6844		2.68E+00			2.6844
	0.1342		1.34E-01			0.1342
	0		0.00E+00			0

	2.1792	2.18E+00	2.1792
	17.4339	1.74E+01	17.4339
	26.1509	2.62E+01	26.1509
	8.717	8.72E+00	8.717
	0.4358	4.36E-01	0.4358

$Y` = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\theta + \sum_{n=1}^{\infty} b_n \sin n\theta$ = 3.459103 cos θ - 3.663936 sinθ + 3.675931 cos2θ - 0.2580795 sin2θ + 2.302111 cos3θ + 1.566614 sin3θ + 0.4792288 cos 4θ +2.435408 sin 4θ + 0.254272 cos 5θ - 3.162009 sin 5θ +2.374001 cos 6θ - 1.281018 sin 6θ +1.966705 cos 7θ + 0.902719 sin 7θ +0.6103806 cos 8θ +2.075833 sin 8θ +1.751606 cos 9θ - 4.125383 sin 9θ +3.679075 cos 10θ- 1.267496 sin 10θ + 3.264007 cos 11θ + 1.168737 sin11θ + 1.586286 cos12θ + 3.297034 sin12θ + 1.986715 cos13θ - 1.926015 sin13θ + 2.322663 cos 14θ - 0.1309131 sin 14θ + 2.164267 cos 15θ - 1.256695 sin 15θ +0.7058266 cos 16θ + 3.047954 sin 16θ +2.482546 cos 17θ - 3.740283 sin 17θ +4.849097 cos 18θ - 0.000000000000189 sin 18θ

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Table 7. Forecasts by Derived Fourier Model ($\theta = 12^0$) Continuous

Y = F5	*Y'*
0.5780531	4.884947
0.6534513	1.792964
0.7288495	-1.84832
0.8042477	-2.17779
1.105841	0.721642
1.181239	-2.62141
1.256637	-0.67554
1.332035	1.931633
1.784425	-3.16574
1.859823	2.259511
1.935221	3.321446
2.010619	-0.23703
2.463009	2.635369
2.538407	-0.04397
2.613805	-2.30203
2.689203	-1.74171
3.066195	-3.1873
3.141593	-3.48158

SUMMARY OF ALGORITHM

a. $R \in (0,1)$

b. $X = -\alpha^{-1}$ in R

c. $B^*(\overline{Z}, \overline{v} \mid v')$ discrete optimal mixed strategy

d. $B^*(\overline{Z}, \overline{v} \mid v')$ continuous optimal mixed strategy

e. $F_5 = \left(\frac{In\ R}{\alpha}\right)^{1/\beta}$

f. Y values of spectral model (output from optimal mixed strategies)

g. $\bar{Y}$ values of spectral model of original data (R)

h. $Y-\bar{Y}$ prediction errors (plot graph)

i. YB Trade-offs of relative frequency of Y versus optimal mixed strategy

   i. $YB_D = B_D - p(Y)$ discrete

   ii. $YB_C = B_C - p(Y)$ continuous

   Y' = values of derived model

Then test for significance of differences using t-test statistics.

1. Prepare table ($R, X, B_D, B_C, F_5, Y, YB$)
2. Plot graph of values
3. Conduct spectral analysis of
   a. F vs. $B^*(\bar{Z}, \bar{v} \mid v')$
   b. F vs. F_5
   c. $B^*(\bar{Z}, \bar{v} \mid v')$ vs. F_5
   d. Y vs. $B^*(\bar{Z}, \bar{v} \mid v')$
   e. Compute trade-offs then interpret results

DISCUSSION

The values in the result were obtained through the computation of equations involving F_i^{-1} (i = 1, 2, 3, 4, 5), having started with an initially generated random number R from the interval (0,1). To compute the next values of F_i^{-1}, the R before this stage are to be used in the equation, so that

R = F_{i-1}^{-1} is plugged into equation for computing F_i^{-1}. The output was tabulated as shown in table 1.

From the analysis of the model of N-Team DM with bounded rationality constraints, it was found that the imposed constraints on the last subordinate DM were colossal. It was found to be N^N, where N is the number of DMS. These constraints were inhibitors to performance in the process. This was the expectation of this simulation.

The results here indicated that there was no consistent decline in the level of performance of subordinate DM as depicted on plotted graph (see Graph 1). The results also indicated fluctuation, which appeared as sinusoidal waves shown in figures 2-5. All the figures are typical of the general profile observed.

The pay-off , $p(Y) - B^*$,yields 55.6 % insignificant difference, while the absolute value of the pay – off , $|p(Y) - B^*|$, yields 79.6 % concordance for the discrete distribution. The pay-off , $p(Y) - B^*$,yields 63.0 % insignificant difference, while the absolute value of the pay – off , $|p(Y) - B^*|$, yields 81.5 % concordance for the continuous distribution.

It can then be concluded that the choice of optimal strategies was appropriate and the derived models have good fit.

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CONCLUSION

The values obtained, using the Weibull distribution, indicated the associated reliability or level of performance of the subordinate/subsystem. The extreme value $F_i^{-1} > 1$, was reciprocated to obtain random deviates for the next stages of the process of command and control, either as SITREP, command input or feedback. There was no consistent decline in the level of performance (reliability, workload, and forecast) of the least/last DM as the result indicated fluctuations, which may be due to choice of initiating parameters. Spectral analysis was use to obtain the model for sinusoidal outputs. The trade-off given by YB indicated that the choice of optimal strategies was appropriate, which confirms the goodness of fit of the model obtained.

This model could become suitable for forecasting performance of the system (C^3I). It could be automated and thus upgraded to C^4I system (computerized command control communication and intelligence). The model could be adopted to predict required input needed for specified outcome/performance/reliability.

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