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THE EFFECT OF INVOLVING MORE DERIVATIVE PROPERTIES, ON THE ACCURACY OF AN IMPLICIT ONE-STEP METHOD

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ABSTRACT

This work describes the development of a class of implicit one-step multi-derivative methods of the form;

$$\sum_{j=0}^{1} \alpha_j \, y_{n+j} = \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j} \,, \quad \alpha_1 = +1$$

with local truncation error

$$T_{n+1} = \sum_{j=0}^{1} \alpha_j \, y_{n+j} - \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j}$$

Also, it investigates the pattern of the order of accuracy of the methods with varying order of derivative. The development of the methods adopts the Taylor Series expansion of the functions y_{n+j}, y'_{n+j}, y''_{n+j}, ---- . Accuracy of order P is imposed on T_{n+1} and the resulting equations are solved for parameters α_j's and β_{ij}'s to generate the required methods (schemes). The order of accuracy and error constants of the methods are determined by substituting the results of parameters α_j's and β_{ij}'s into the original equations. The methods were implemented by using them to solve some initial value problems of first order ordinary differential equations in order to establish the pattern of the accuracy. The result showed that the order of accuracy of the methods increased as the order of the derivative increased with optimum accuracy obtained at l =3.

Keywords: Implicit, Multi-derivative, One-step, Order of accuracy, Derivative property

INTRODUCTION

According to Burden (2004), many problems encountered in the various branches of Science, Engineering and Management give rise to differential equations which can either be ordinary or partial (O .D. E. or P. D .E.). O.D.Es. occur when the dependent variable y is a function of a single independent variable x e.g $\frac{dy}{dx} = x - y^2$. Partial differential equations occur when the dependent variable v is a function of two or more independent variables, x and y e.g $y\frac{\partial v}{\partial x} + x\frac{\partial v}{\partial y} = 0$.

Various approximation methods for solving ordinary differential equations have been developed, they include; implicit family of linear multistep methods by the Adams (Hairer

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Vol. 2 No. 3 September 2013

Ernst and Gerhard Wanner, 1996), defined as: $\sum_{j=0}^{k} \alpha_j \, y_{n+j} = h \sum_{j=0}^{k} \beta_j \, y'_{n+j}$, backward difference formula by Fatunla, (1988), explicit Runge-kutta method of the form; $y_{n+1} - y_n = h\emptyset\,(x_n, y_n, h)$, Famurewa *et al,* (2011), implicit family of linear multiderivative multistep methods by Famurewa (2011), defined as $\sum_{j=0}^{1} \alpha_j \, y_{n+j} = \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j}$, $\alpha_1 = +1$ and so on.

Explicit one-step method is the easiest to implement, it requires no additional starting values and it readily permits a change of step-length during the computation (Lambert, 1991). Its low order of accuracy of course makes it of limited practical value. This is why linear multistep methods achieve higher order by sacrificing the one-step nature of the algorithm while retaining linearity with respect to $y_{n+j}, y^i_{n+j}, \ldots$.

This study will show that higher order of accuracy can be achieved in a one-step method, by involving more analytical properties of the differential equation by way of more of the derivative properties of y (Famurewa *et al.*, 2011).

This study will consider the development of a class of implicit one-step methods, for which the order of the derivative (l) is varied from $l = 1,2,\ldots 5$. Also, attempt will be made to determine the effect of increasing the order of derivative, on the order of accuracy of the one-step methods.

DERIVATION OF THE METHODS

A class of linear one-step methods of the form:

$$\sum_{j=0}^{1} \alpha_j \, y_{n+j} = \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j}, \quad \alpha_1 = +1 \qquad \text{- - - - - - - - - - (1.1)}$$

is developed by using the local truncation error formula:

$$T_{n+1} = \sum_{j=0}^{1} \alpha_j \, y_{n+j} - \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j} \qquad \text{- - - - - - - - - - (1.2)}$$

to determine parameters α_j's and β_{ij}'s for $l = 1,2,\ldots,5$.

Consequently, it was assumed that the local truncation error T_{n+1} for one-step application of the formula to equation (1.1) can be defined as;

$$T_{n+1} = \sum_{j=0}^{1} \alpha_j \, y_{n+j} - \sum_{i=1}^{l} h^i \sum_{j=0}^{1} \beta_{ij} \, y^i{}_{n+j}$$

Adopting the Taylor series expansion of $y^i{}_{n+j}$, $j = 0(1)$, defined by

$$y^i{}_{n+j} = \sum_{r=0}^{\infty} \frac{(jh)^r \, y^{r+i}}{r!}, \quad i = 0(1)l \qquad \text{(Barrio, 2006)}$$

in equation (1.2) and combine terms in equal powers of h to have:

$$T_{n+1} = C_o y_n + C_1 h y_n' + C_2 h^2 y_n'' \ldots = \sum_{i=0}^{\infty} C_i h^i y_n{}^i \qquad \ldots\ldots\ldots \qquad (1.3)$$

where

$$C_1 = \sum_{j=1}^{k} j\alpha_j - \sum_{j=0}^{k} \beta_{1j}$$

$$C_2 = \frac{1}{2!}\sum_{j=1}^{k} j^2\alpha_j - \sum_{j=1}^{k} j\beta_{1j} - \sum_{j=0}^{k} j\beta_{2j}$$

$$C_3 = \frac{1}{3!}\sum_{j=1}^{k} j^3\alpha_j - \frac{1}{2!}\sum_{j=1}^{k} j^2\beta_{1j} - \sum_{j=0}^{k} j\beta_{2j}$$

$$C_4 = \frac{1}{4!}\sum_{j=1}^{k} j^4\alpha_j - \frac{1}{3!}\sum_{j=1}^{k} j^3\beta_{1j} - \frac{1}{2!}\sum_{j=0}^{k} j^2\beta_{2j}$$

$$C_5 = \frac{1}{5!}\sum_{j=1}^{k} j^5\alpha_j - \frac{1}{4!}\sum_{j=1}^{k} j4\beta_{1j} - \frac{1}{3!}\sum_{j=0}^{k} j^3\beta_{2j}$$

$$C_P = \frac{1}{P!}\sum_{j=1}^{k} j^P\alpha_j - \frac{1}{(P-1)!}\sum_{j=1}^{k} j^{(P-1)}\beta_{1j} - \frac{1}{(P-2)!}\sum_{j=1}^{k} j^{(P-2)}\beta_{2j}$$

and

$$C_{P+1} = \frac{1}{(P+1)!}\sum_{j=1}^{k} j^{P+1}\alpha_j - \frac{1}{P!}\sum_{j=1}^{k} j^P\beta_{1j} - \frac{1}{(P-1)!}\sum_{j=1}^{k} j^{(P-1)}\beta_{2j} \quad - \quad - \quad -(1.4)$$

Setting $l=1$ in equation (1.1) gives

$$\alpha_0 y_n + \alpha_1 y_{n+1} = h\beta_{10}\, y^{(1)}{}_n + h\beta_{11}\, y^{(1)}{}_{n+1} \quad - \quad - \quad - \quad (1.5)$$

with local truncation error

$$T_{n+1} = \alpha_0 y_n + \alpha_1 y_{n+1} - h\beta_{10}\, y^{(1)}{}_n - h\beta_{11}\, y^{(1)}{}_{n+1} \quad - \quad - \quad (1.6)$$

Adopting the Taylor's expansion of y_{n+1} and $y^1{}_{n+1}$ given as

$$y_{n+1} = y_n + hy_n{}^{(1)}{}_n + \frac{h^2 y_n{}^{(11)}}{2!} + \frac{h^3 y_n{}^{(111)}}{3!} + \ldots + 0(h^4)$$

$$y^1{}_{n+1} = y^{(1)}{}_n + hy_n{}^{(11)} + \frac{h^2 y_n{}^{(111)}}{2!} + \frac{h^3 y^{(IV)}}{3!} + \ldots + 0(h^4)$$

in equation (1.6) and combining terms in equal powers of h gives

$$T_{n+1} = (\alpha_0 + \alpha_1)y_n + (\alpha_1 - \beta_{10} - \beta_{11})hy^{(1)}_n + \left(\frac{\alpha_1}{2} - \beta_{11}\right)h^2 y^{(11)}_n + \left(\frac{\alpha_1}{6} - \frac{\beta_{11}}{2}\right)h^3 y^{(111)}_n + 0(h^4) \quad - \ (1.7)$$

that is;

$$T_{n+1} = C_0 y_n + C_1 hy^{(1)}_n + C_2 h^2 y^{(11)}_n + C_3 h^3 y^{(111)}_n + \ldots + 0(h^4)$$ where

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Vol. 2 No. 3 September 2013

$$\left.\begin{array}{l} C_0 = \alpha_0 + \alpha_1 \\ C_1 = \alpha_1 - \beta_{10} - \beta_{11} \\ C_2 = \frac{\alpha_1}{2} - \beta_{11} \\ C_3 = \frac{\alpha_1}{6} - \frac{1}{2}\beta_{11} \end{array}\right\} \quad - \quad - \quad - \quad - \quad - \quad (1.8)$$

Imposing accuracy of order 2 on T_{n+1} to have $C_0 = C_1 = C_2 = 0$ and $T_{n+1} = 0(h^3)$ that is;

$$\begin{array}{l} \alpha_0 + \alpha_1 = 0 \\ \alpha_1 - \beta_{10} - \beta_{11} = 0 \\ \frac{1}{2}\alpha_1 - \beta_{11} = 0 \end{array} \qquad \text{where } \alpha_1 = +1$$

gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \end{pmatrix}$$

Solving gives

$\alpha_0 = -1$, $\beta_{10} = ½$, and $\beta_{11} = ½$.

putting these values into equation (1.5) gives the one – step, first derivative method of the form;

$$y_{n+1} = y_n + \frac{h}{2}(y_{n+1}^{(1)} + y_n^{(1)}) \quad - \quad - \quad - \quad - (1.9)$$

which coincides with the trapezoidal rule.

Setting l=2 in equation (1.1) gives

$$\alpha_0 y_n + \alpha_1 y_{n+1} = h\left(\beta_{10} y^{(1)} n + \beta_{11} y^{(1)}{}_{n+1}\right) + h^2\left(\beta_{20} y^{(11)}_n + \beta_{21} y^{(11)}_{n+1}\right) - \quad - \qquad (1.10)$$

with local truncation error

$$T_{n+1} = \alpha_0 y_n + \alpha_1 y_{n+1} - h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) - h^2\left(\beta_{20} y_n^{(11)} + \beta_{21} y_{n+1}^{(11)}\right) \quad - \quad - \quad (1.11)$$

Adopting Taylor's expansion of $y_{n+1}, y_{n+1}^{(1)}$ *and* $y_{n+1}^{(11)}$ in equation (1.11) and combining terms in equal powers of h gives

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Vol. 2 No. 3 September 2013

$$T_{n+1} = (\alpha_0 + \alpha_1)y_n + (\alpha_1 - \beta_{10} - \beta_{11})hy_n^{(1)} + \left(\frac{\alpha_1}{2} - \beta_{11} - \beta_{20}\right)h^2 y_n^{(11)} + \left(\frac{\alpha_1}{6} - \frac{\beta_{11}}{2} - \beta_{21}\right)h^3 y_n^{(111)}$$
$$+ \left(\frac{\alpha_1}{24} - \frac{\beta_{11}}{6} - \frac{\beta_{21}}{2}\right)h^4 y_n^{(IV)} + \left(\frac{\alpha_1}{120} - \frac{\beta_{11}}{24} - \frac{\beta_{21}}{6}\right)h^5 y_n^{(V)} + 0(h^6) \qquad - \quad - \quad - (1.12)$$

that is

$$T_{n+1} = C_0 y_n + C_1 hy_n^{(1)} + C_2 h^2 y_n^{(11)} + C_3 h^3 y_n^{(111)} + \ldots + 0(h^4)$$

where

$$\left.\begin{aligned} C_0 &= \alpha_0 + \alpha_1 \\ C_1 &= \alpha_1 - \beta_{10} - \beta_{11} \\ C_2 &= \tfrac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta_{21} \\ C_3 &= \frac{1}{6}\alpha_1 - \tfrac{1}{2}\beta_{11} - \beta_{21} \\ C4 &= \frac{1}{24}\alpha_1 - \frac{1}{6}\beta_{11} - \frac{1}{2}\beta_{21} \\ C5 &= \frac{\alpha_1}{120} - \frac{\beta_{11}}{24} - \frac{\beta_{21}}{6} \end{aligned}\right\} \qquad - \quad - \quad - \quad - \quad - (1.13)$$

Imposing accuracy of order 5 on T_{n+1} to have

$C_0 = C_1 = C_2 = C_3 = C_4 = 0$ and $T_{n+1} = 0(h^5)$

This gives

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & -\frac{1}{2} & 0 & -1 \\ 0 & 0 & -\frac{1}{6} & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \\ \beta_{20} \\ \beta_{21} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \\ -\frac{1}{6} \\ -\frac{1}{24} \end{pmatrix}$$

Solving gives

$$\alpha_0 = -1,\ \beta_{10} = \tfrac{1}{12},\ \beta_{11} = \tfrac{1}{2}\ \ \beta_{20} = \tfrac{1}{12}\ \ and\ \ \beta_{21} = -\tfrac{1}{12}$$

Putting these values into equation (1.10) gives a one – step, second derivative method of the form;

$$y_{n+1} = y_n + \frac{h}{2}\left(y^{(1)}_{n+1} + y^{(1)}_n\right) - \frac{h^2}{12}\left(y^{(11)}_{n+1} - y^{(11)}_n\right) \ldots (1.14)$$

Setting l=3 in equation (1.1) gives

$$\alpha_0 y_n + \alpha_1 y_{n+1} = h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) + h^2\left(\beta_{20} y_n^{(11)} + \beta_{21} y_{n+1}^{(11)}\right) + h^3\left(\beta_{30} y_n^{(111)} + \beta_{31} y_{n+1}^{(111)}\right) ..(1.15)$$

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Vol. 2 No. 3 September 2013

with local truncation error

$$T_{n+1} = \alpha_0 y_n + \alpha_1 y_{n+1} - h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) - h^2\left(\beta_{20} y_n^{(11)} + \beta_{21} y_{n+1}^{(11)}\right) - h^3\left(\beta_{30} y_n^{(111)} + \beta_{31} y_{n+1}^{(111)}\right) \quad (1.16)$$

Adopting Taylor's expansion of $y_{n+1}, y_{n+1}^{1}, y_{n+1}^{11}$ *and* y_{n+1}^{111} in equation (1.1) and combining terms in equal powers of h gives

$$T_{n+1} = (\alpha_0 + \alpha_1) y_n + (\alpha_1 - \beta_{10} - \beta_{11}) h y_n^{(1)} + \left(\frac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta_{21}\right) h^2 y_n^{(11)}$$
$$+ \left(\frac{1}{3!}\alpha_1 - \frac{1}{2}\beta_{11} - \beta_{21} - \beta_{30} - \beta_{31}\right) h^3 y_n^{(111)} + \left(\frac{1}{4!}\alpha_1 - \frac{1}{3!}\beta_{11} - \frac{1}{2}\beta_{21} - \beta_{31}\right) h^4 y_n^{(IV)}$$
$$+ \left(\frac{1}{5!}\alpha_1 - \frac{1}{4!}\beta_{11} - \frac{1}{3!}\beta_{21} - \frac{1}{2}\beta_{31}\right) h^5 y_n^{(V)} + \left(\frac{1}{6!}\alpha_1 - \frac{1}{5!}\beta_{11} - \frac{1}{4!}\beta_{21} - \frac{1}{3!}\beta_{31}\right) h^6 y_n^{(VI)}$$
$$+ \left(\frac{1}{7!}\alpha_1 - \frac{1}{6!}\beta_{11} - \frac{1}{5!}\beta_{21} - \frac{1}{4!}\beta_{31}\right) h^7 y_n^{(VII)} + 0(h^8) \qquad - \qquad - \qquad (1.17)$$

That is

$$T_{n+1} = C_0 y_n + C_1 h y_n^{(1)} + |C_2 h^2 y_n^{(11)} + C_3 h^3 y_n^{(111)} + C^4 h^4 y_n^{(IV)} \ldots + 0(h^5)$$

Where

$$\left.\begin{aligned}
C_0 &= \alpha_0 + \alpha_1 \\
C_1 &= \alpha_1 - \beta_{10} - \beta_{11} \\
C_2 &= \frac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta_{21} \\
C_3 &= \frac{1}{3!}\alpha_1 - \frac{1}{2}\beta_{11} - \beta_{21} - \beta_{30} - \beta_{31} \\
C_4 &= \frac{1}{4!}\alpha_1 - \frac{1}{3!}\beta_{11} - \frac{1}{2}\beta_{21} - \beta_{31} \\
C_5 &= \frac{1}{5!}\alpha_1 - \frac{1}{4!}\beta_{11} - \frac{1}{3!}\beta_{21} - \frac{1}{2}\beta_{31} \\
C_6 &= \frac{1}{6!}\alpha_1 - \frac{1}{5!}\beta_{11} - \frac{1}{4!}\beta_{21} - \frac{1}{3!}\beta_{31} \\
C_7 &= \frac{1}{7!}\alpha_1 - \frac{1}{6!}\beta_{11} - \frac{1}{5!}\beta_{21} - \frac{1}{4!}\beta_{31}
\end{aligned}\right\} \qquad - \quad - \quad - \qquad (1.18)$$

Imposing accuracy of order 7 on T_{n+1} to have

$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 0$ and $T_{n+1} = 0(h^7)$

That is

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Vol. 2 No. 3 September 2013

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & -1 & -10 & -1 \\ 0 & 0 & -\frac{1}{6} & 0 & -\frac{1}{2} & 0 & -1 \\ 0 & 0 & -\frac{1}{24} & 0 & -\frac{1}{6} & 0 & --\frac{1}{2} \\ 0 & 0 & -\frac{1}{120} & 0 & -\frac{1}{24} & 0 & -\frac{1}{6} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \\ \beta_{20} \\ \beta_{21} \\ \beta_{30} \\ \beta_{31} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \\ -\frac{1}{6} \\ -\frac{1}{24} \\ -\frac{1}{120} \\ -\frac{1}{720} \end{pmatrix}$$

Solving gives

$\alpha_0 = -1$, $\beta_{10} = 0.5$, $\beta_{11} = 0.5$, $\beta_{20} = 0.1$, $\beta_{21} = -0.1$, $\beta_{30} = 0.0083$ and $\beta_{31} = 0.0083$.

Putting these values into equation (1.15) gives the one – step third derivative method of the form;

$$y_{n+1} = y_n + \frac{h}{2}\left(y_{n+1}^{(1)} + y_n^{(1)}\right) - \frac{h^2}{10}\left(y_{n+1}^{(11)} - y_n^{(11)}\right) + \frac{h^3}{120}\left(y_{n+1}^{(111)} + y_n^{(111)}\right) - - - \quad (1.19)$$

Setting l=4 in equation (1.1) gives

$$\alpha_0 y_n + \alpha_1 y_{n+1} = h\left(\beta_{10} y^{(1)} n + \beta_{11} y^{(1)}{}_{n+1}\right) - h^2\left(\beta_{20} y^{(11)}{}_n + \beta_{21} y_{n+1}^{(11)}\right) - h^3\left(\beta_{30} y^{(111)}{}_n + \beta_{31} y_{n+1}^{(111)}\right) - h^4\left(\beta_{40} y_n^{(IV)} + \beta_{41} y_{n+1}^{(IV)}\right) \quad ------(1.20)$$

With local truncation error

$$T_{n+1} = \alpha_0 y_n + \alpha_1 y_{n+1} - h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) - h^2\left(\beta_{20} y^{(11)}{}_n + \beta_{21} y_{n+1}^{(11)}\right) - h^3\left(\beta_{30} y_n^{(111)} + \beta_{31} y_{n+1}^{(111)}\right) - h^4\left(\beta_{40} y_n^{(IV)} + \beta_{41} y_{n+1}^{(IV)}\right) - - - - - \quad (1.21)$$

Adopting Taylor's expansion of $y_{n+1}, y_{n+1}^{1}, y_{n+1}^{11}, y_{n+1}^{111}$ and y_{n+1}^{IV} in equation (1.21) and combining terms in equal powers of h gives

$$\begin{aligned} T_{n+1} &= (\alpha_0 + \alpha_1)y_n + (\alpha_1 - \beta_{10} - \beta_{11})hy_n^{(1)} + \left(\frac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta^{21}\right)h^2 y_n^{(11)} \\ &+ \left(\frac{1}{3!}\alpha_1 - \frac{1}{2}\beta_{11} - \beta_{21} - \beta_{30} - \beta_{31}\right)h^3 y_n^{(111)} + \left(\frac{1}{4!}\alpha_1 - \frac{1}{3!}\beta_{11} - \frac{1}{2!}\beta_{21} - \beta_{31} - \beta_{40} - \beta_{41}\right)h^4 y_n^{(IV)v} \\ &+ \left(\frac{1}{5!}\alpha_1 - \frac{1}{4!}\beta_{11} - \frac{1}{3!}\beta_{21} - \frac{1}{2!}\beta_{31} - \beta_{41}\right)h^5 y_n^{(V)} + \left(\frac{1}{6!}\alpha_1 - \frac{1}{5!}\beta_{11} - \frac{1}{4!}\beta_{21} - \frac{1}{3!}\beta_{31} - \frac{1}{2!}\beta_{41}\right)h^6 y_n^{(VI)} \\ &+ \left(\frac{1}{7!}\alpha_1 - \frac{1}{6!}\beta_{11} - \frac{1}{5!}\beta_{21} - \frac{1}{4!}\beta_{31} - \frac{1}{3!}\beta_{41}\right)h^7 y_n^{(VII)} + \left(\frac{1}{8!}\alpha_1 - \frac{1}{7!}\beta_{11} - \frac{1}{6!}\beta_{21} - \frac{1}{5!}\beta_{31} - \frac{1}{4!}\beta_{41}\right)h^8 y_n^{(VIII)} \\ &+ \left(\frac{1}{9!}\alpha_1 - \frac{1}{8!}\beta_{11} - \frac{1}{7!}\beta_{21} - \frac{1}{6!}\beta_{31} - \frac{1}{5!}\beta_{41}\right)h^9 y_n^{(IX)} + 0(h^{10}) \quad - - - \quad (1.22) \end{aligned}$$

that is

$$T_{n+1} = C_0 y_n + C_1 h y_n^{(1)} + |C_2 h^2 y^{(11)}{}_n + C_3 h^3 y_n^{(111)} + C^4 h^4 y_n^{(IV)} \ldots + 0(h^5)$$

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Vol. 2 No. 3 September 2013

where

$$\left.\begin{aligned}
C_0 &= \alpha_0+\alpha_1 \\
C_1 &= \alpha_1 - \beta_{10} - \beta_{11} \\
C_2 &= \tfrac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta_{21} \\
C_3 &= \frac{1}{3!}\alpha_1 - \frac{1}{2}\beta_{11} - \beta_{21} - \beta_{30} - \beta_{31} \\
C_4 &= \frac{1}{4!}\alpha_1 - \frac{1}{3!}\beta_{11} - \frac{1}{2}\beta_{21} - \beta_{31} - \beta_{40} - \beta_{41} \\
C_5 &= \frac{1}{5!}\alpha_1 - \frac{1}{4!}\beta_{11} - \frac{1}{3!}\beta_{21} - \frac{1}{2!}\beta_{31} - \beta_{41} \\
C_6 &= \frac{1}{6!}\alpha_1 - \frac{1}{5!}\beta_{11} - \frac{1}{4!}\beta_{21} - \frac{1}{3!}\beta_{31} - \frac{1}{2!}\beta_{41} \\
C_7 &= \frac{1}{7!}\alpha_1 - \frac{1}{6!}\beta_{11} - \frac{1}{5!}\beta_{21} - \frac{1}{4!}\beta_{31} - \frac{1}{3!}\beta_{41} \\
C_8 &= \frac{1}{8!}\alpha_1 - \frac{1}{7!}\beta_{11} - \frac{1}{6!}\beta_{21} - \frac{1}{5!}\beta_{31} - \frac{1}{4!}\beta_{41} \\
C_9 &= \frac{1}{9!}\alpha_1 - \frac{1}{8!}\beta_{11} - \frac{1}{7!}\beta_{21} - \frac{1}{6!}\beta_{31} - \frac{1}{5!}\beta_{41}
\end{aligned}\right\} \quad - \quad - \qquad (1.23)$$

Imposing accuracy of order 8 on T_{n+1} to have

$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = 0$ and $T_{n+1} = 0(h^9)$

which gives

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{2} & 0 & -1 & -1 & -1 & 0 & 0 \\
0 & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 & -1 & -1 \\
0 & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 \\
0 & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} \\
0 & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} \\
0 & 0 & -\frac{1}{7!} & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!}
\end{pmatrix}
\begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \\ \beta_{20} \\ \beta_{21} \\ \beta_{30} \\ \beta_{31} \\ \beta_{40} \\ \beta_{41} \end{pmatrix}
=
\begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \\ -\frac{1}{3!} \\ -\frac{1}{4!} \\ -\frac{1}{5!} \\ -\frac{1}{6!} \\ -\frac{1}{7!} \\ \frac{1}{8!} \end{pmatrix}$$

solving gives

$\alpha_0 = -1$, $\beta_{10} = 0.5$, $\beta_{11} = 0.5$, $\beta_{20} = 0.1071$, $\beta_{21} = -0.1071$, $\beta_{30} = 0.0119$, $\beta_{31} = 0.0119$

$\beta_{40} = 0.0006$, $\beta_{41} = -0.0006$.

Putting these values into equation (1.20) gives the one – step, fourth derivative method of the form;

$$y_{n+1} = y_n + 0.5h\left(y_{n+1}^{(1)} + y_n^{(1)}\right) - 0.11h^2\left(y_{n+1}^{(11)} - y_n^{(11)}\right) + 0.01h^3\left(y_{n+1}^{(111)} + y_n^{(111)}\right) \quad \text{- - (1.24)}$$

Setting *l*=5 in equation (1.1) gives

$$\alpha_0 y + \alpha_1 y_{n+1} = h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) + h^2\left(\beta_{20} y_n^{(11)} + \beta_{21} y_{n+1}^{(11)}\right) + h^3\left(\beta_{30} y_n^{(111)} + \beta_{31} y_{n+1}^{(111)}\right) + h^4\left(\beta_{40} y_n^{(IV)} + \beta_{41} y_{n+1}^{(IV)}\right) + h^5\left(\beta_{50} y_n^{(V)} + \beta_{51} y_{n+1}^{(V)}\right) \quad --(1.25)$$

with local truncation error

$$T_{n+1} = \alpha_0 y_n + \alpha_1 y_{n+1} - h\left(\beta_{10} y_n^{(1)} + \beta_{11} y_{n+1}^{(1)}\right) - h^2\left(\beta_{20} y_n^{(11)} + \beta_{21} y_{n+1}^{(11)}\right) - h^3\left(\beta_{30} y_n^{(111)} + \beta_{31} y_{n+1}^{(111)}\right) - h^4\left(\beta_{40} y_n^{(IV)} + \beta_{41} y_{n+1}^{(IV)}\right) - h^5\left(\beta_{50} y_n^{(V)} + \beta_{51} y_{n+1}^{(V)}\right) \quad - - - - - (1.26)$$

Adopting Taylor's expansion of $y_{n+1}, y_{n+1}^{1}, y_{n+1}^{11}, y_{n+1}^{111}, y_{n+1}^{IV}$ *and* y_{n+1}^{V} in equation (1.26) and combining terms in equal powers of h gives

$$T_{n+1} = C_0 y_n + C_1 h y_n^{(1)} + C_2 h^2 y_n^{(11)} + C_3 h^3 y_n^{(111)} + \ldots C_{11} h^{11} y_n^{(XI)} + 0(h^{12}) \quad - - - (1.27)$$

where

$$C_0 = \alpha_0 + \alpha_1$$

$$C_1 = \alpha_1 - \beta_{10} - \beta_{11}$$

$$C_2 = \frac{1}{2}\alpha_1 - \beta_{11} - \beta_{20} - \beta_{21}$$

$$C_3 = \frac{1}{3!}\alpha_1 - \frac{1}{2}\beta_{11} - \beta_{21} - \beta_{30} - \beta_{31}$$

$$C_4 = \frac{1}{4!}\alpha_1 - \frac{1}{3!}\beta_{11} - \frac{1}{2!}\beta_{21} - \beta_{31} - \beta_{40} - \beta_{41}$$

$$C_5 = \frac{1}{5!}\alpha_1 - \frac{1}{4!}\beta_{11} - \frac{1}{3!}\beta_{21} - \frac{1}{2!}\beta_{31} - \beta_{41} - \beta_{50} - \beta_{51}$$

$$C_6 = \frac{1}{6!}\alpha_1 - \frac{1}{5!}\beta_{11} - \frac{1}{4!}\beta_{21} - \frac{1}{3!}\beta_{31} - \frac{1}{2!}\beta_{41} - \beta_{51}$$

$$C_7 = \frac{1}{7!}\alpha_1 - \frac{1}{6!}\beta_{11} - \frac{1}{5!}\beta_{21} - \frac{1}{4!}\beta_{31} - \frac{1}{3!}\beta_{41} - \frac{1}{2!}\beta_{51}$$

$$C_8 = \frac{1}{8!}\alpha_1 - \frac{1}{7!}\beta_{11} - \frac{1}{6!}\beta_{21} - \frac{1}{5!}\beta_{31} - \frac{1}{4!}\beta_{41} - \frac{1}{3!}\beta_{51}$$

$$C_9 = \frac{1}{9!}\alpha_1 - \frac{1}{8!}\beta_{11} - \frac{1}{7!}\beta_{21} - \frac{1}{6!}\beta_{31} - \frac{1}{5!}\beta_{41} - \frac{1}{4!}\beta_{51}$$

$$C_{10} = \frac{1}{10!}\alpha_1 - \frac{1}{9!}\beta_{11} - \frac{1}{8!}\beta_{21} - \frac{1}{7!}\beta_{31} - \frac{1}{6!}\beta_{41} - \frac{1}{5!}\beta_{51}$$

$$C_{11} = \frac{1}{11}\alpha_1 - \frac{1}{10!}\beta_{11} - \frac{1}{9!}\beta_{21} - \frac{1}{8!}\beta_{31} - \frac{1}{7!}\beta_{41} - \frac{1}{6!}\beta_{51}. \quad ---------(1.28)$$

Imposing accuracy of order 10 on T_{n+1} gives

$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 0$ and $T_{n+1} = 0(h^{11})$

ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

that is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 & 0 & -1 \\ 0 & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} & 0 & -1 \\ 0 & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{7!} & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} & 0 & -\frac{1}{3!} \\ 0 & 0 & -\frac{1}{8!} & 0 & -\frac{1}{7!} & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} & 0 & -\frac{1}{4!} \\ 0 & 0 & -\frac{1}{9} & 0 & -\frac{1}{8!} & 0 & -\frac{1}{7!} & 0 & -\frac{1}{6!} & 0 & -\frac{1}{5!} \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \beta_{10} \\ \beta_{11} \\ \beta_{20} \\ \beta_{21} \\ \beta_{30} \\ \beta_{31} \\ \beta_{40} \\ \beta_{41} \\ \beta_{50} \\ \beta_{51} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -\frac{1}{2} \\ -\frac{1}{3!} \\ -\frac{1}{4!} \\ -\frac{1}{5!} \\ -\frac{1}{6!} \\ -\frac{1}{7!} \\ -\frac{1}{8!} \\ -\frac{1}{9} \\ -\frac{1}{10} \end{pmatrix}$$

Solving gives

α_0 = -1, β_{30} = 0.0139, β_{10} = 0.5, β_{31} = 0.0139, β_{11} = 0.5, β_{40} = 0.0010, β_{20} = 0.1111, β_{41} = 0.0000

β_{21} = 0.1111, β_{50} = 0.0000 and β_{51} = 0.0000.

Putting these values into equation (1.25) gives a one – step fifth derivative method of the form;

$$y_{n+1} = y_n + 0.5h\left(y_{n+1}^{(1)} + y_n^{(1)}\right) - 0.1h^2\left(y_n^{(11)} + y_n^{(11)}\right) + 0.01h^3(y_{n+1}^{(111)} + y_n^{(111)}) \text{..(1.29)}$$

ANALYSIS OF ACCURACY OF THE METHODS

According to Butcher (2008); Onumanyi *et al.*, (2008); & Famurewa (2011), a numerical method is accurate if and only if order P of the truncation error T_{n+k} of the method is greater than or equal to one, (P ≥ 1).

For the first derivative method ($l = 1$),

$\alpha_0 = -1$, $\beta_{10} = 1/2$ and $\beta_{11} = 1/2$

Substituting for the values of these parameters in equation (1.8) we have

$$C_0 = -1 + 1 = 0$$

$$C_1 = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

$$C_2 = \frac{1}{2} - \frac{1}{2} = 0$$

$$C_3 = \frac{1}{6} - \frac{1}{4} = \frac{-1}{12}$$

that is

$$C_0 = C_1 = C_2 = 0, \ C_3 \neq 0$$

ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

Hence, the order P = 2 with error constant $C_{P+1} = \frac{-1}{12}$

For the second derivative method ($l = 2$);

$$\alpha_0 = -1, \quad \beta_{10} = 0.5, \quad \beta_{11} = 0.5, \quad \beta_{20} = 0.0833 \; and \; \beta_{21} = -0.0833$$

Substituting for the values of these parameters in equation (1.13) we have

$$C_0 = -1 + 1 = 0$$

$$C_1 = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

$$C_2 = \frac{1}{2} - \frac{1}{2} - 0.0833 + 0.0833 = 0$$

$$C_3 = \frac{1}{6} - \frac{1}{4} + 0.0833 = 0$$

$$C_4 = \frac{1}{24} - \frac{1}{12} + \frac{0.0833}{2} = 0$$

$$C_5 = \frac{1}{120} - \frac{1}{48} + \frac{0.0833}{6} = \frac{1}{720}$$

that is

$$C_0 = C_1 = C_2 = C_3 = C_4 = 0, C_5 \neq 0 \quad ,$$

Hence, the order P = 4 with error constant $C_{P+1} = \frac{1}{720}$

For the third derivative method($l = 3$),

$$\alpha_0 = -1, \beta_{10} = 0.5, \beta_{11} = 0.5, \beta_{20} = 0.1, \beta_{21} = -0.1, \beta_{30} = 0.0083 \; and \; \beta_{31} = 0.0083$$

Substituting for the values of these parameters in equation (1.18) we have

$$C_0 = -1 + 1 = 0$$

$$C_1 = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

$$C_2 = \frac{1}{2} - \frac{1}{2} - \frac{1}{10} + \frac{1}{10} = 0$$

$$C_3 = \frac{1}{6} - \frac{1}{4} + \frac{1}{10} - \frac{1}{120} - \frac{1}{120} = 0$$

$$C_4 = \frac{1}{24} - \frac{1}{12} + \frac{1}{20} - \frac{1}{120} = 0$$

$$C_5 = \frac{1}{120} - \frac{1}{48} + \frac{1}{60} - \frac{1}{240} = 0$$

$$C_6 = \frac{1}{720} - \frac{1}{240} + \frac{1}{240} - \frac{1}{720} = 0$$

ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

$$C_7 = \frac{1}{5040} - \frac{1}{1440} + \frac{1}{1200} - \frac{1}{2880} = \frac{-1}{2880}$$

that is

$$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = 0, C_7 \neq 0$$

Hence, the order P = 6 with error constant $C_{P+1} = \frac{-1}{2880}$

For the fourth derivative method ($l = 4$),

$\alpha_0 = -1,\ \beta_{10} = 0.5,\ \beta_{11} = 0.5,\ \beta_{20} = 0.1071,\ \beta_{21} = -0.1071,\ \beta_{30} = 0.0119,$ $\beta_{31} = 0.0119,\ \beta_{40} = 0.0006\ and\ \beta_{41} = -0.0006.$

Substituting for the values of these parameters in equation (1.23) gives

$$C_0 = -1 + 1 = 0$$

$$C_1 = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

$$C_2 = 0.5 - 0.5 - 0.1071 + 0.1071 = 0$$

$$C_3 = \frac{1}{6} - \frac{1}{4} + 0.1071 - 0.0119 - 0.0119 = 0$$

$$C_4 = \frac{1}{24} - \frac{1}{12} + \frac{1}{2}(0.1071) - 0.0119 - 0.0006 + 0.0006 = 0$$

$$C_5 = \frac{1}{120} - \frac{1}{48} + \frac{1}{6}(0.1071) - \frac{1}{2}(0.0119) + 0.0006 = 0$$

$$C_6 = \frac{1}{720} - \frac{1}{240} + \frac{1}{24}(0.1071) - \frac{1}{6}(0.0119) + \frac{1}{2}(0.0006) = 0$$

$$C_7 = \frac{1}{5040} - \frac{1}{1440} + \frac{1}{120}(0.1071) - \frac{1}{24}(0.0119) + \frac{1}{6}(0.0006) = 0$$

$$C_8 = \frac{1}{40320} - \frac{1}{10080} + \frac{1}{720}(0.1071) - \frac{1}{120}(0.0119) + \frac{1}{24}(0.0006) = 0$$

$$C_9 = \frac{1}{362880} - \frac{1}{80640} + \frac{1}{5040}(0.1071) - \frac{1}{720}(0.0119) + \frac{1}{120}(0.0006) = 7.7160*10\text{^}{-8}$$

$$C_1 = C_2 = C_3 =,,,,= C_8 = 0\ and\ C_9 = 7.7160 * 10\text{^} - 8$$

Hence, the order P = 8 with error constant C_{P+1} = 7.7160*10^-8

For the fifth derivative method ($l = 5$),

$\alpha_0 = -1,\ \beta_{10} = 0.5,\ \beta_{11} = 0.5, \beta_{20} = 0.1111,\ \beta_{21} = -0.1111,\ \beta_{30} = 0.0139,\ \beta_{31} = 0.0139, \beta_{40} = 0.0010,\ \beta_{41} = -0.0000,\ \beta_{50} = 0.0000$ and $\beta_{51} = 0.0000.$

Substituting for the values of these parameters in equation (1.28) we have

$$C_0 = -1 + 1 = 0$$

ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

$$C_1 = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

$$C_2 = \frac{1}{2} - \frac{1}{2} - 0.1111 + 0.1111 = 0$$

$$C_3 = \frac{1}{6} - \frac{1}{2}\left(\frac{1}{2}\right) + 0.1111 - 0.0139 - 0.0139 = 0$$

$$C_4 = \frac{1}{24} - \frac{1}{6}\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)0.1111 - 0.0139 - 0.0010 = 0$$

$$C_5 = \frac{1}{120} - \frac{1}{24}\left(\frac{1}{2}\right) + \left(\frac{1}{6}\right)0.1111 - \left(\frac{1}{2}\right)0.0139 = 0$$

$$C_{10} = \frac{1}{10!} - \frac{1}{9!}\left(\frac{1}{2}\right) + \left(\frac{1}{8!}\right)0.1111 - \left(\frac{1}{7!}\right)0.0139 = 0$$

$$C_{11} = \frac{1}{11!} - \frac{1}{10!}\left(\frac{1}{2}\right) + \left(\frac{1}{9!}\right)0.1111 - \left(\frac{1}{8!}\right)0.0139 = -1.5131 * 10^{\wedge} - 7$$

Hence, the order P = 10 with error constant $C_{P+1} = -1.5131 * 10^{\wedge} - 7$

Since the condition for accuracy is satisfied by all these methods, then the methods are accurate.

IMPLEMENTATION

To access the accuracy of the schemes, the methods were re-written in FORTRAN programming language and implemented on a digital computer.

The program was used to solve two (2) initial value problems of Ordinary Differential Equation;

1. $y^{i} = x + y\ ,\ y(0) = 1,\ x \in [0,1]$ with $h = 0.1$
2. $y^{i} = -10(y - x^3)\ , y(0) = 1,\ x \in [0,1]$ with $h = 0.1$

The results and errors are shown in Tables 1 - 4.

DISCUSSION OF RESULTS

A numerical method is said to be accurate when the order P of the local truncation error T_{n+k} is greater than or equal to one ($P \geq 1$). From the analysis of the accuracy of the methods, it was discovered that the first derivative method has order P = 2, the second derivative method has order P =4, the third derivative method has order P = 6, the fourth derivative method has order P = 8 and the fifth derivative method has order P =10. Hence all the methods are accurate. The pattern of accuracy was observed with the results obtained from the solution of the I.V.Ps, that the accuracy of the schemes increased from l = 1 to l = 3 and decreased from l =4 to l = 5. Hence, one-step multi-derivative methods have optimal accuracy at l = 3.One-step third-derivative method is therefore recommended for solving stiff and non-stiff I.V.Ps.

CONCLUSION

In this study, a class of implicit linear one-step multi-derivative methods has been developed for solution of ordinary differential equations. Analysis of the accuracy showed that the methods are accurate, suggesting that the accuracy of a one-step method can be improved by introducing more derivative properties into the method.

ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

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ISSN: 2186-8476, ISSN: 2186-8468 Print
Vol. 2 No. 3 September 2013

Table 1. Results for Problem 1

X	*Exact-Solution*	*L=1 Computed Solution*	*L=2 Computed-Solution*	*L=3 Computed-Solution*	*L=4 Computed-Solution*	*L=5 Computed-Solution*
0.1	1.110341836 1512953	1.110500000 0000000	1.110341798 6111110	1.110341836 1514121	1.110327819 2061594	1.105927819 2061599
0.2	1.242805516 3203395	1.243155125 0000001	1.242805433 3437009	1.242805516 3205978	1.242774534 1783285	1.233058771 7738222
0.3	1.399717615 1520065	1.400297201 9062501	1.399717477 5969552	1.399717615 1524342	1.399666254 4713636	1.383576015 2097225
0.4	1.583649395 2825408	1.584503482 4068829	1.583649192 5867531	1.583649395 2831714	1.583573712 4150751	1.559887493 4527696
0.5	1.797442541 4002564	1.798622473 9302075	1.797442261 3833965	1.797442541 4011277	1.797337988 6016190	1.764649081 7639273
0.6	2.044237600 7810177	2.045802489 3113618	2.044237229 4212330	2.044237600 7821732	2.044098942 7657206	2.000790114 7044458
0.7	2.327505414 9409531	2.329523201 3113827	2.327504936 1222502	2.327505414 9424426	2.327326634 5674135	2.271541542 5745382
0.8	2.651081856 9849350	2.653630518 2494058	2.651081252 2117915	2.651081856 9868166	2.650856048 7071366	2.580466986 9490597
0.9	3.019206222 3138993	3.022375130 2951558	3.019205470 3890037	3.019206222 3162388	3.018925471 8696947	2.931496993 8125586
1.0	3.436563656 9180902	3.440455112 7587211	3.436562733 5786235	3.436563656 9209630	3.436218905 5376859	3.328966813 5310269
1.1	3.908332047 8928663	3.913063013 3765765	3.908330925 4001568	3.908332047 8963581	3.907912939 1022045	3.777658070 7972487

Table 2. Results for Problem 2

X	*Exact-Solution*	*L=1 Computed-Solution*	*L=2 Computed-Solution*	*L=3 Computed-Solution*	*L=4 Computed-Solution*	*L=5 Computed-Solution*
0.1	0.368879441 1714423	0.251999999 9999999	0.365147916 6666666	0.368238010 4398148	0.372303226 789058	0.152304764 4873561
0.2	0.143335283 2366127	0.071750000 0000000	0.138894552 9513889	0.140822646 5940407	0.143669324 827529	0.026516172 4167677
0.3	0.076787068 3678639	0.043937500 0000000	0.070940722 4301939	0.071265952 1009912	0.072548063 846543	0.019359709 0918280
0.4	0.082315638 8887342	0.069234375 0000000	0.073844013 3860082	0.072711389 7686226	0.072872246 921344	0.044277599 8109633
0.5	0.131737946 9990855	0.127308593 7500000	0.119536879 8803155	0.117009463 4247452	0.116400868 740101	0.093208955 0226451
0.6	0.218478752 1766663	0.217577148 4375000	0.201598862 4563650	0.197601142 5037185	0.196399247 636303	0.170017718 4569627
0.7	0.343911881 9655544	0.344394287 1093749	0.321501668 6038830	0.315867640 0122182	0.314131187 471123	0.280379391 6978305
0.8	0.512335462 6279024	0.513348571 7773436	0.483594358 3451656	0.476110322 1919935	0.473835234 409056	0.430244678 6552882
0.9	1.000045399 9297620	0.730337142 9443357	0.693275026 4800081	0.683704488 9542892	0.680856280 527271	0.625606090 5270335
1.0	0.729123409 8040864	1.001334285 7360836	0.956324228 4041693	0.944420396 7190106	0.940950682 545484	0.872462489 6536708
1.1	1.331016701 7007897	1.332333571 4340205	1.278661958 2723531	1.264173553 5618808	1.260027826 313018	1.176813703 2079780

Table 3. Error for Problem 1

X-Value	*L=1* *Error*	*L=2* *Error*	*L=3* *Error*	*L=4* *Error*	*L=5* *Error*
0.1000	1.581638e-004	3.754018e-008	1.167955e-013	1.401695e-005	4.414017e-003
0.2000	3.496087e-004	8.297664e-008	2.582379e-013	3.098214e-005	9.746745e-003
0.3000	5.795868e-004	1.375551e-007	4.276579e-013	5.136068e-005	1.614160e-002
0.4000	8.540871e-004	2.026958e-007	6.306067e-013	7.568287e-005	2.376190e-002
0.5000	1.179933e-003	2.800169e-007	8.713030e-013	1.045528e-004	3.279346e-002
0.6000	1.564889e-003	3.713598e-007	1.155520e-012	1.386580e-004	4.344749e-002
0.7000	2.017786e-003	4.788187e-007	1.489475e-012	1.787804e-004	5.596387e-002
0.8000	2.548661e-003	6.047731e-007	1.881606e-012	2.258083e-004	7.061487e-002
0.9000	3.168908e-003	7.519249e-007	2.339462e-012	2.807504e-004	8.770923e-002
1.0000	3.891456e-003	9.233395e-007	2.872813e-012	3.447514e-004	1.075968e-001
1.1000	4.730965e-003	1.122493e-006	3.491873e-012	4.191088e-004	1.306740e-001

Table 4. Error for Problem

X-Value	*L=1* *Error*	*L=2* *Error*	*L=3* *Error*	*L=4* *Error*	*L=5* *Error*
0.1000	1.168794e-001	3.731525e-003	6.414307e-004	3.423786e-003	2.165747e-001
0.2000	7.158528e-002	4.440730e-003	2.512637e-003	3.340416e-004	1.168191e-001
0.3000	3.284957e-002	5.846346e-003	5.521116e-003	4.239005e-003	5.742736e-002
0.4000	1.308126e-002	8.471626e-003	9.604249e-003	9.443392e-003	3.803804e-002
0.5000	4.429353e-003	1.220107e-002	1.472848e-002	1.533708e-002	3.852899e-002
0.6000	9.016037e-004	1.687989e-002	2.087761e-002	2.207950e-002	4.846103e-002
0.7000	4.824051e-004	2.241021e-002	2.804424e-002	2.978069e-002	6.353249e-002
0.8000	1.013109e-003	2.874110e-002	3.622514e-002	3.850023e-002	8.209078e-002
0.9000	1.213733e-003	3.584838e-002	4.541892e-002	4.826713e-002	1.035173e-001
1.0000	1.288886e-003	4.372117e-002	5.562500e-002	5.909472e-002	1.275829e-001
1.1000	1.316870e-003	5.235474e-002	6.684315e-002	7.098888e-002	1.542030e-001